

MASTER THESIS

Analysis of the impact of transaction costs on the performance of Bull ETFs

By

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Abstract

Since 2008 both Handelsbanken and DnbNOR have offered the Bull funds on the Oslo stock exchange. These are funds whose shares trade like stocks on the stock exchange, and the fund provide investors with twice the daily return of the OBX-index. In order to do so, the fund managers utilize OBX-stock index futures contracts. Fund managers take long futures positions of twice the value of the fund to achieve the double exposure. To maintain the double exposure, fund managers must rebalance the fund at the end of each trading day. They do this by adjusting the number of futures contracts so that the exposure is always twice that of the funds value. The transaction costs associated with rebalancing is deducted from the value of the fund. This master thesis investigates if such transaction costs affect the performance of the Bull fund, and whether the magnitude of the transaction costs effect on performance might depend on the level of expected return and standard deviation of the benchmark stock index.

Our results are based on data from simulation runs, using MATLAB. First, we compute and compare M-squared performance measures with and without transaction costs with the stock index as the benchmark. M-squared performance measures are computed for 6 time horizons ranging from daily to yearly. We find that the Bull fund generally underperforms the benchmark both with and without transaction costs, but that the underperformance is always greater with transaction costs. The M-squared performance, both with and without transaction costs, also show a decrease in performance for both higher values of the benchmark's expected return and standard deviation. Comparing M-squared measures reveals that differences in performance become greater against higher values of the benchmark's standard deviation. Difference in M-squared measures also shows a decrease in differences, for longer time horizons, against higher values of the benchmark's expected return. Simulation data is also used to estimate expected returns and standard deviation, with 95% confidence intervals, for daily, weekly and monthly time horizons. Analysis shows that standard deviations are not significantly different for any of these time horizons, while the expected return of weekly and monthly time horizons are significantly reduced by transaction costs. However, we are not able to explain the increase in M-squared difference against higher values of the benchmarks standard deviation and the decrease in M-squared differences for higher values of the

benchmark's expected return, by analyzing the estimated expected returns and standard deviations.

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Chapter 1: Introduction

In 2008 both Handelsbanken and DnB NOR introduced the Bull ETFs on the Oslo stock exchange. These are so-called exchange traded funds [ETFs]; simply meaning that they are funds, managed by the institutions, whose shares trade on a stock exchange like stocks. As the “Bull” name indicates, these are funds for investors who have a positive market-sentiment. The object of both funds is to provide twice the daily return of the OBX stock index which is made up of the 25 most liquid stocks on the Oslo stock exchange. In order to achieve twice the daily return of the index, the funds utilizes OBX-index futures and take long futures positions worth twice the value of the fund, with the value of the fund mostly being made up of cash holdings. Similar funds have been offered in the American market since 2006, where they are referred to as Leveraged ETFs (Ferri, 2009). The leverage or “gearing-effect” offered by the Bull funds means that the double daily returns comes at the price of high volatility, double that the benchmark index. Both providers of the Bull fund in the Norwegian market have marketed their funds as high-risk investment objects, most suitable for short term investment horizons. Handelsbanken rates their fund’s risk profile as 5 out of 5 (Handelsbanken kapitalforvaltning 2008) while Dnb NOR rates their Bull fund’s risk profile as 9 out of 10 (Dnbnor Kapitalforvaltning AS, 2008). The way the Bull funds work is that if the daily return of the OBX-index is 1%, the return on the Bull fund will be 2%, and if the return of the OBX-index is -1%, then the return on the Bull fund is -2%. In order to always maintain a double exposure to the OBX-index, the Bull funds must be *rebalanced* at the end of each trading day: After the futures positions of the fund have been marked-to-market, the fund manager has to adjust the number of OBX-index futures contracts so that the exposure is once again twice that of the value of the fund.

The daily rebalancing of the fund means that transaction costs accrue on a daily basis. These are costs which are deducted from the value of the fund and are spread equally among the outstanding shares, held by investors. The objective in this paper is therefore to investigate how transaction costs affect the performance of the Bull fund. Further we also investigate whether the magnitude of the effect of transaction costs on performance might vary with the levels of expected returns and standard deviations/volatility of the benchmark stock index. As previously stated, the volatility of a Bull fund is related to the volatility of the underlying benchmark. The volatility of the Leveraged ETFs is much discussed in the available research

papers on the topic, and high realized volatility is often showed to negatively affect returns (Avellaneda, & Zhang, 2009). Co (2009) models the return of a Leveraged ETF and shows how the return is negatively related to the realized volatility of the benchmark, while Hill & Foster (2009) finds that a Leveraged ETF has a higher probability of providing double the daily returns over a longer period of time if the volatility of the underlying benchmark is low.

The research problems are formulated:

- *If any, what are the effects of transaction costs on the performance of a Bull fund?*
- *If any, does the magnitude of such effects depend on the characteristics, i.e. the size of expected return and standard deviation, of the benchmark?*

To try to answer these problems we utilize MATLAB to run price path simulations of a benchmark stock index and two Bull funds; one with and one without transaction costs. The transaction costs are represented as the bid-ask spread of futures contracts and management fees. The data from the simulations are then used to compute M-squared performance measures for the two funds, using the underlying stock index as the benchmark. The data is also used to compute the estimated expected returns and standard deviations with 95% confidence intervals. All these performance measures and estimates are computed for different values of the annual expected return and annual standard deviation of the benchmark. The M-squared measures are calculated for daily, weekly, monthly, quarterly, semi-annual and annual time horizons, while the estimated expected returns and standard deviations are calculated for daily, weekly and monthly time horizons. In trying to answer the research problems we compare the M-squared measures, estimated expected returns and standard deviations of the fund with and without transaction costs.

Because the Bull funds are marketed as short term investment vehicle we choose only to focus the analysis of estimated expected returns and standard deviations for short term investment horizons, i.e. daily, weekly and monthly. At the same time we choose to use the M-squared measures to capture any general trends from a daily time horizons up to an annual time horizon.

Analyzing the M-squared measures shows that the Bull fund actually underperforms the benchmark for all periods except daily, even without transaction costs. The negative performances generally increase for higher values of both expected return and standard deviation of the benchmark. The M-squared measures with transaction costs are negative for all time horizons, and also exhibits a general decrease in performance for higher values of the benchmark's expected return and standard deviation.

We find through the comparison of M-squared measures that the fund with transaction costs underperforms the fund without transaction costs for all time horizons and values of expected return and standard deviation. In comparing the estimated daily, weekly and monthly expected returns and standard deviations we find that transaction costs causes both the expected returns and standard deviations to decline, but that the decline in expected return is greater than the decline in standard deviation. This causes the daily, weekly and monthly M-square measures to be less with transaction cost than without. However, the estimated daily, weekly and monthly standard deviations are not statistically significantly different with and without transaction costs. The weekly and monthly expected returns are all statistically significantly less with transaction costs, while the daily estimated expected returns are not significantly different.

In trying to answer the second research problem our results are inconclusive. The difference between the M-squared measure without transaction costs and the M-squared measure with transaction costs increases for higher values of the benchmark's standard deviation, for all time horizons. However, when we analyze and compare the estimated daily, weekly and monthly expected returns and standard deviations we find no clear connection with the increased M-square difference for these three periods. The monthly, quarterly, semi-annual and annual difference between the M-squared measures without and the M-squared measure with transaction cost show small declines in performance difference for higher values of the benchmark's expected return. The analysis and comparison of estimated monthly expected returns and standard deviations offer no clear answer to why this might be.

The rest of this master thesis is organized in the following way: Chapter 2 gives an introduction to the Bull funds. The chapter explains how futures contracts work, how the Bull fund is constructed and attributes of the performance of the Bull fund. Chapter 3 gives a thorough review of investment transaction costs and how they affect investments. The end of the chapter deals with the actual transaction costs of the Bull fund. Chapter 4 contains the theory regarding futures pricing, assumptions of stock index movements and performance measures. Chapter 5 reviews the current papers and work written on leveraged ETFs and Bull funds. Chapter 6 outlines the methods and models used for simulations in detail. Chapter 7 contains the analysis of results and chapter 8 contains the conclusion. Chapter 9 contains the bibliography. The Appendix contains the data from simulation runs stored in tables and the codes for the MATLAB scripts used for simulations.

Chapter 2: What is Leveraged Exchange traded funds?

Leveraged Exchange traded funds [LETF] are basically funds whose shares trade like stocks on a stock exchange. The purpose of a LETF is to provide twice, (and sometimes three-times) the daily return of the benchmark index, at twice (or three-times) the daily volatility of the benchmark, making LETFs a risky investment asset. There are type main types of LETFs: The *long* and the *short*. The long LETF is designed to provide twice the daily return, as discussed above. The short LETF is designed to provide twice the opposite daily return of the benchmark index. This means that is the daily return of the benchmark is negative; the short LETF will have a positive daily return, twice the size of the negative daily return.

2.1 LETFs in the Norwegian Market

There are two providers of LETFs on the Oslo Stock Exchange [OSE]: Handelsbanken and DnB NOR. They both offer one long and one short LETF each. The long LETFs have been nicknamed *Bull* and the short ones are nicknamed *Bear*. The funds have traded on the OSE since 2008. These funds are all benchmarked to the OBX-total return index, and are designed to give twice or minus twice, the daily return of the index. The fund managers of both institutions exclusively use OBX-futures contracts, in order to achieve the designated daily returns. The following section gives a general description of futures contracts. It is necessary in order to give a meaningful explanation of how the Bull fund works.

2.2 Futures contracts

Futures contracts are not assets themselves, but rather contracts for trading a certain asset in the future. By entering a futures contract, one can either agree to buy or sell an asset at some time in the future, for a price agreed upon today. That price is known as the *futures price*. Buying is known as going *long* and selling is known as going *short*, a futures contract. What makes futures contracts special is that they are standardized and that they trade on organized exchanges. The standardization of the futures contracts is drawn up by the exchange at which the futures contract trades. Standardization encompasses many elements with the most notable being, *asset*, *contract size* and *time of delivery*. Futures contracts are drawn up for many types of assets like agricultural products, oil, currencies, interest rates and stock indexes. The contract size describes the amount of the asset that is being traded for each contract. The time of delivery describes when the described amount of an asset is set to be delivered. This is also

known as the *maturity*- or *delivery date* of the futures contract. The delivery date of futures contracts are often quoted in months, with delivery taking place on a certain day, for example the third Friday of the month. Most futures contracts do not however go to delivery. Investors will in most cases *close out* their futures positions. If one has 10 long stock index futures position and wishes to close out, one would need to take 10 short positions in the same futures contract. This will mean that your obligations are nullified. Investors with short position close out by taking an offsetting number of long positions. Closing out is possible because all futures exchanges have a *clearing house* which monitors, matches and keeps track of all futures and obligations. Clearing houses are discussed in detail in its own section (Hull, 2008).

2.2.1 Stock index futures contracts and the investors who use them

Stock index futures are futures contracts on the underlying portfolio of a stock index. The contract size of stock index futures is given as the futures price times a set multiple to quote the contract size in monetary terms. Stock index futures are settled in cash, rather than delivery of the underlying portfolio. Cash settlement is both more practical and cost efficient than actually making delivery of the underlying portfolio of stocks. This point becomes quite clear if one considers the time spent and transaction costs associated with buying and making delivery of all 500 hundred stocks that makes up the underlying portfolio of the S&P 500, for a single futures contract.

Stock index futures have become popular because of the cost effective way it allows for exposure against stock indexes and for the ease of which positions can be close out. The popularity also means that investors do not have to worry about getting their futures positions matched, as there is always investors who wish to take the long or short position if the price is right. There are three types of investors that utilize futures contracts in general: *Speculators*, *hedgers* and *arbitrageurs*. Speculators will use stock index futures to gamble on the market movements. Through the use of futures they are able to time their entry and exit from the market in a cost effective and swift manner, far more superior than actually trading in the stocks of the underlying portfolio. Hedger use futures contracts to manage risk. Suppose an investor is holding the underlying portfolio of stocks that make up a stock index. The investor is bullish but expects a short term bear market in the following period. He has the option of

selling off his stock positions and placing the proceeds in risk-free assets. When the bear market is over he can then buy back his stock positions. However, this strategy comes at the expense of transaction costs. Hedging with stock index futures offers a more cost effective alternative. By holding his original stock portfolio and entering short stock index futures, the investor hedges his position against the fall in the market. His portfolio will be reduced in value, but the short positions futures will compensate for the loss. When the market turns again, the hedger can easily close out the short positions. Arbitrageurs use discrepancies between futures price and spot price of the underlying asset to earn risk-free profits. If the futures price is too high the arbitrageurs will take short stock index positions and at the same time buy the portfolio of stocks underlying the index. If the futures price is too low the arbitrageurs will take long futures positions and sell the stocks of the underlying portfolio. Arbitrageurs use *program trading* to execute coordinated trades. Program trading involves using computers for programming in trades that are sent and executed at the same time. Price discrepancies cannot be expected to last long so arbitrageurs are forced to act quickly (Bodie, Kane, & Marcus, 2008, p.821-826). In chapter 4 we will show how the theoretical futures price is derived under the condition of no-arbitrage opportunities.

2.2.2 Futures contract payoffs

The payoff of any futures contract is linked with the spot price of the asset underlying the futures contract. At the delivery date of a futures contract the futures price must be equal to the spot price of the same asset. If this is not the case then it is possible to buy or sell an asset at two different prices. Such conditions cannot be expected to last, as investors will jump on the opportunity to buy the asset at the lowest price and sell it at the highest price, until the two prices are equal. The convergence of futures and spot prices at maturity is called *the convergence principle* (Bodie, Kane, & Marcus, 2008, p.793).

The payoff of a long and short position is demonstrated:

The following notations are used:

T = Delivery/maturity-date time T

$F_{(0,T)}$ = Futures price at time zero with maturity at time T

$S_T = F_T$ = Spot price at time T equal to futures price at time T

Payoff of long futures position at time T

The payoff of a long futures position is given as:

$$(S_T - F_{(0,T)}) = Long\ Payoff \quad (2.1)$$

Figure 2.1-The payoff of a long futures position at maturity

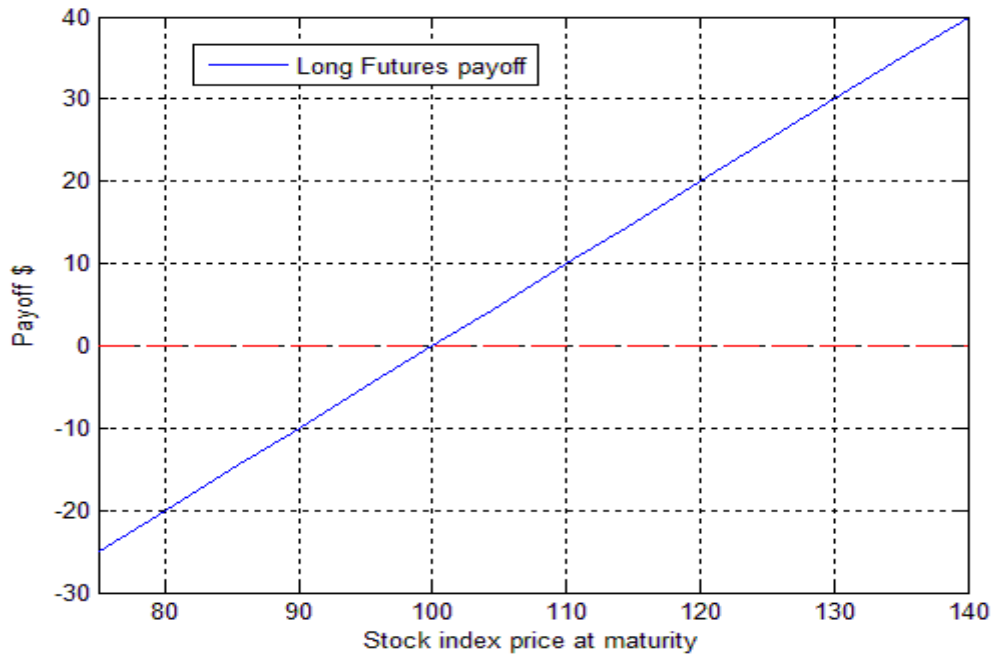


Figure 2.1 shows the graphical illustration of the long futures payoff at maturity. $F_{(0,T)}=100$ and S_T goes from 75 to 140. For $S_T > 100$ the payoff is positive and for $S_T < 100$ the payoff is negative. If $S_T = 100$ the payoff is equal to zero. If the futures contract in question is a stock index, the total payoff at maturity, will be:

$$(Stock\ index\ value\ at\ maturity - Futures\ stock\ index\ price) * multiple(\$) = Payoff$$

Assume that: $S_T=120$ $F_{(0,T)}=100$ and $multiple = 250\$$

$$(120 - 100) * 250\$ = 5000\$$$

Payoff of a short futures position at time T

The payoff of a short futures position is given as:

$$(F_{(0,t)} - S_T) = Short\ Payoff \quad (2.2)$$

Figure 2.2-The payoff of a short futures position at maturity

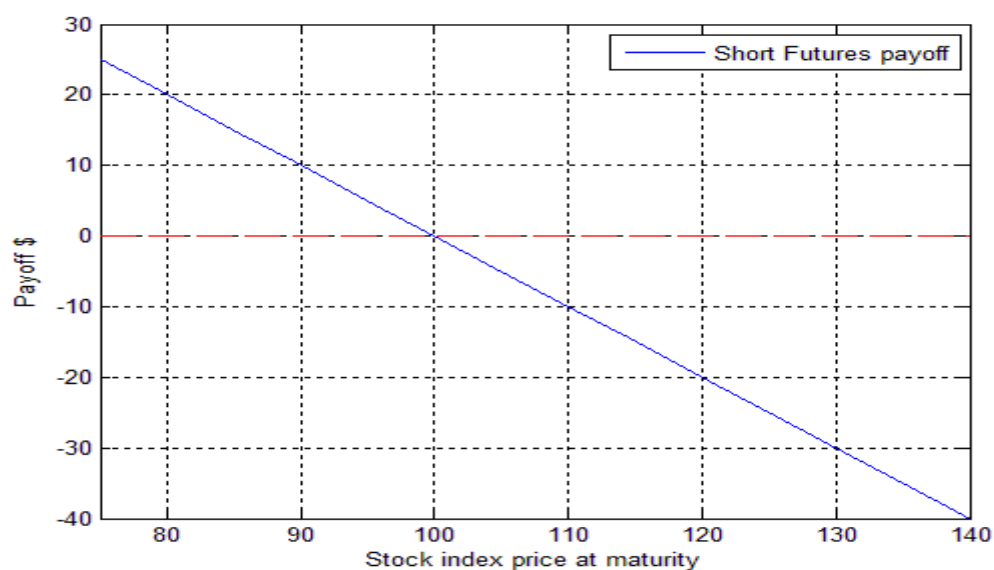


Figure 2.2 illustrates the payoff of a short futures position at maturity. $F_{(0,T)} = 100$ and S_T goes from 75 to 140. For $S_T > 100$ the short payoff is negative and for $S_T < 100$ the payoff is positive. If $S_T = 100$ the payoff is zero. The total payoff, at maturity, of a short stock index futures contract:

$$(Futures\ stock\ index\ price - Stock\ index\ value\ at\ maturity) * multiple(\$) = Short\ payoff$$

Assume that: $S_T=120$ $F_{(0,T)}=100$ and $multiple = 250\$$

$$(100 - 120) * 250\$ = -\$5000$$

2.2.3 How futures contracts are traded

Investors trade in futures contracts through brokers on the exchange. Brokers will seek to find other investors who are willing to take the opposite side of a contract. Today, matching of orders are often done with the help of electronics. Brokers simply input desired trades which are matched when a counterpart emerges. Because futures contracts are standardized; the only elements of negotiation are the futures price, and the number of contracts that an investors wishes to trade. When trades are matched, the futures price is effectively locked in. This is now the price which will be paid at the delivery date in the future (Hull, 2008).

2.2.4 Marking-to-market and margin accounts

Because futures contracts are contracts for future trades, they are effectively agreements between two parties. In order to protect the parties from default on either side, both have to post *margins* on a *margin account*. Margins are a certain sum of money that futures investors have to post on their margin accounts. The margin accounts are kept by the investor's broker. The margin account comes in to play at the end of a trading day, when all futures contracts are *marked-to-market*. The futures price of certain type of futures contracts reflects the supply and demand for the contract. As such, the futures price will often move away from the futures price for which a trade was matched. The price movement represents a gain for one party and a loss for the other. When a futures contract is marked-to-market, the futures price is marked to the closing price, and the daily gains and losses of the two parties will be settled at their margin account. The party with an unfavorable price movement will have money deducted from his account while the party for which the price movement was favorable will have money deposited on his account. The exact amount for a single contract is equal to the difference between the futures price and the closing price times the contract size. When a contract is marked-to-market the next day, it is marked using the closing price from the day before against the current closing price. If the amount on the margin account falls beneath a limit, known as the *maintenance margin*, the broker will instruct the investor to deposit more funds. The extra funds deposited are known as the *variation margin*. If the investor fails to post a variation margin, the broker will close out the investor's position. It must be noted that investors actually earn interest on their margin deposits, as such posting a margin does not represent a cost (Hull, 2008, p.26-27). The following example shows how marking-to-market works for a stock index futures contract.

Example: Suppose the futures price of a matched stock index futures contract is $F_0 = 100$ at time zero and that the multiple is $M = \text{NOK } 100$. This means that the total contract size of one stock index futures contract is $F_0 * M = 100 * \text{NOK } 100 = \text{NOK } 10.000$. Assume that the brokers of the contract demands that the parties of the futures contract must post 10% to the total contract value as margin. This means that the long and short parties of the contract must post $10.000 \text{ NOK} * 10\% = 1.000 \text{ NOK}$ each per contract on their margin accounts. The maintenance margin per account is set to 600 NOK.

At the end of day 1 the futures price has risen to $F_1 = 105$ and the contract is marked-to-market. This rise in the futures price represents a gain for the long party and a loss for the short party. The total gain of the long party is $(F_1 - F_0) * M = (105 - 100) * 100 \text{ NOK} = 500 \text{ NOK}$. The total loss of the short party is $(F_0 - F_1) * M = (100 - 105) * 100 \text{ NOK} = -500 \text{ NOK}$. The broker will therefore deposit 500 NOK on the margin account of the long party, bringing the total balance to $1.000 \text{ NOK} + 500 \text{ NOK} = 1.500 \text{ NOK}$. At the same time the broker will also deduct 500 NOK from the short party's margin account bringing the balance to $1.000 \text{ NOK} - 500 \text{ NOK} = 500 \text{ NOK}$. The balance on the short party's margin account is now below the maintenance margin. The short party is therefore instructed to post a variation margin or the broker will close out the short party's futures position. The short party post a 500 NOK variation margin bringing the balance on his margin account back to $500 \text{ NOK} + 500 \text{ NOK} = 1.000 \text{ NOK}$.

At the end of day 2 the stock index futures contract is once again marked-to-market. The futures price has gone down and is equal to $F_2 = 95$. This reduction of the futures price represents a loss for the long party and a gain for the short party. The total loss of the long party is $(F_2 - F_1) * M = (95 - 105) * 100 \text{ NOK} = -1.000 \text{ NOK}$. The total gain of the short party is $(F_1 - F_2) * M = (105 - 95) * 100 \text{ NOK} = 1.000 \text{ NOK}$. The short party will have the gain deposited to his margin account bringing the balance to $(1000 \text{ NOK} + 1000 \text{ NOK}) = 2000 \text{ NOK}$. The long party will have the loss deducted from his margin account, bringing the balance to $(1.500 \text{ NOK} - 1.000 \text{ NOK}) = 500 \text{ NOK}$. The balance of the long party's margin account is now below the maintenance margin. The long party will therefore have to post a variation margin if he does not want his broker to close out the long futures position.

As previously stated, investors are free to close out their futures positions themselves, by taking offsetting futures positions. In the advent of closing out the gains or losses of a futures investor will be given by the difference between the futures price at investment and the futures price when the investor closed out times the contract size and the number of contracts (Hull,2008,p.21-22).

2.2.5 The role of the clearinghouse

Every futures exchange has a designated *clearinghouse*. The role of the clearinghouse is to keep tabs on all traded futures contracts and to guarantee for the performance of both parties. A clearinghouse deals exclusively with members of the clearing house, known as *clearing members*. If a broker is not a member, he must channel his business through one of the clearing members (Hull,2008,p.28). Once a futures contract is matched, the clearing house becomes the intermediary between the two parties of a contract, making it the seller to the long position and buyer to the short position (Bodie, Kane, & Marcus, 2008,p.790). By placing itself in the middle it, the clearinghouse effectively guarantees for the performance of both clearing members. In order to guarantee for the performance of both parties, the clearinghouse demands that clearing members post a *clearing margin*. Like the margin-account of the investor, the clearing margin is subject to adjustments of daily gains and losses. The amount to be posted as a clearing margin is equal to an *original margin* times the number of traded futures contracts. Broker who are not clearing members, must keep a margin account with clearing member it does business with (Hull,2008,p28-29). The adjustment of margin-accounts trickles down from clearing margin-accounts to the investor's margin-account held by the broker.

2.3 The design and mechanics of the Bull fund

The value of a Bull fund is given by the cash holding of the fund. The value per share of the fund is called the *Net Asset Value* [NAV]. The NAV at time t is calculated by:

$$NAV_t = \frac{\sum MV_t + \sum I_t - \sum K_t}{\sum A_t} \quad (2.3)$$

MV_t is the market value of the fund's positions in financial instruments and cash holding at time t . I_t is the accrued, but not mature earning of the fund, while K_t is the debt of the fund and any accrued, but not matures costs at time t . A_t is the number of outstanding shares of the fund at time t . The cash holdings of the fund earn the risk-free interest rate, which is considered as earnings, I_t . Costs are management fees and transaction costs, associated with taking futures positions. The management fee is a percentage of the total value of the fund. It accrues on a daily basis and is deducted from the cash holdings of the fund. At its introduction, DnB NOR set the management fee of the Bull fund to 0.8% p.a. The transaction

costs are associated with the fund manager's trading activity in stock index futures. They include broker commissions, fees and depot costs (Dnbnor Kapitalforvaltning AS, 2008).

The fund manager takes long stock index futures positions worth, twice the value of MV_t , in order to achieve the double exposure to daily returns of the benchmark index. Part of the fund's money is put up as margin. If the futures multiple is 100, the exposure of the Bull fund is given by Haga & Lindset (2009, p.4):

$$EXP_t = \frac{F_{(t,T)} * 100 * N_t}{A_t * NAV_t} \quad (2.4)$$

Where $F_{(t,T)}$ is the price of a stock index futures contract that matures at time T . N_t is the number the fund's futures positions at time t . By rewriting (2.4), the number of futures contracts can be expressed as:

$$N_t = \frac{EXP_t * MV_t}{F_{(t,T)} * 100} \quad \text{where } MV_t = A_t * NAV_t \quad (2.5)$$

The marking-to-market of futures contracts changes the NAV value at the end of every day. In order to maintain a double exposure the fund manager will therefore *re-balance* the fund after the total values of the fund has been calculated. This is done by either take more or closing out long futures positions, so that (2.4) is once again equal to 2.

2.4 Attributes of the performance of the Bull fund

Twice the daily return does not mean that the return will be twice index return over a longer period of time. This is because the daily returns of the fund will be compounded. The holding period returns for periods longer than a day are most likely to diverge from twice the holding period return of the benchmark index. Hill & Foster (2009) shows how the compounded

return of a LETF over a two day period will differ from twice the compounded return of the benchmark:

Example 1: Suppose the benchmark has a positive return daily return of 2% over a two day period. This means that the compounded return of the benchmark is $((1,02 * 1,02) - 1) * 100 = 4,04\%$, making the double of compounded return $(4,04\% * 2) = 8,08\%$. Suppose the Bull fund perfectly replicates the twice the daily returns. The daily return of the Bull fund is therefore 4% each day. This makes the compounded return over the period equal to $((1,04 * 1,04) - 1) * 100 = 8,16\%$ which is greater than twice the period return of the benchmark.

Example 2: Now suppose that the benchmark has a negative daily return of -2% each day over a two day period. The compounded return of the period of the benchmark is $((0,98 * 0,98) - 1) * 100 = -3,96\%$, meaning that twice the negative return over the period is equal to $-3,96\% * 2 = -7,92\%$. The Bull fund successfully replicates twice the daily return of the benchmark, making the return per day equal to -4% . The compounded return over the period is thereby equal to $((0,96 * 0,96) - 1) * 100 = -7,84\%$ which is less than twice the holding period return of the benchmark.

The reason for the difference in returns between the Bull fund and the benchmark is attributable to the daily rebalancing of the fund. In example 1, this means that the fund manager takes more long futures positions when the fund has a positive return after the first day. This amplifies the positive returns of the next day. In example 2, the fund manager closes out the futures positions after the loss of day 1. This reduces the loss of day 2.

Both Handelsbanken and DnB NOR have marketed their Bull funds as high risk investment objects, and rightfully so. Consider the examples again, but suppose that the market turned the other way on the second day. This would mean that the losses of the second day would be amplified in example 1, and that the positive returns would be muffled in example 2. The volatility in the return of the benchmark can be shown to have negative effect on the value of

the bull fund. DnbNOR(2008) gives an example of how the volatility in daily returns over a three day period:

Example 3: This example shows how the volatility negatively affects the return of a Bull fund over a short period of time. The example considers a three day period in which the return of the benchmark goes both up and down, but at the end of day three has a price equal to the price at the beginning of the period. The futures contract multiple is $M = 100$, and the daily payoff from the futures positions are calculated as $(F_{t+1} - F_t) * M * N_t$. N_t is calculated according to (2.5) so that the number of long futures contracts gives double the exposure of MV_t . The results are given in table 2.1. The example shows how the volatility in the returns of the underlying reduces the value of the fund making the holding period returns negative, even though the holding period return of the benchmark is equal to 0%.

Table 2.1- The effect of volatility on the return of the Bull fund.

t	0	1	2	3
F_t	100	102	104	100
Payoff		1.600.000	1.631.400	-3.325.600
MV_t	40.000.000	41.600.000	43.231.400	39.905.800
$MV_t * 2$	80.000.000	83.200.000	86.462.800	79.811.600
N_t	8000	8157	8314	7981
$\frac{F_{t+1} - F_t}{F_t}$		2%	1,96%	-3,85%
$\frac{MV_{t+1} - MV_t}{MV_t}$		4%	3,92%	-7,70%
$\frac{F_3 - F_0}{F_0}$				0%
$\frac{MV_3 - MV_0}{MV_0}$				-0,24%

2.3.1 Other factors of the Bull funds performance

The composition and construction of the benchmark index is also something that investors should make note of. Stock indexes are basically made up of an underlying hypothetical portfolio of stocks. The Bull funds traded at OSE are both benchmarked to the OBX-index. The OBX-index is total return index meaning that it treats the dividends paid on the stocks

that make up the index as reinvested. This means that the index measures the total return on the underlying portfolio. Indexes that do not treat dividends as reinvested are called *price indexes*, meaning that such indexes measures the capital gains or losses on the underlying portfolio, opposed to total return (Bacon 2008, p.40).

Stock indexes also use different techniques for weighting the different stocks that make up the underlying portfolio. The OBX-index uses the most commonly used method: *market-capitalization* weighting. This means that each stock is weighted according to its market-capitalization relative to the total market-capitalization of all stocks included. The market-capitalization of a stock is calculated as number of outstanding shares times the current market price. Sometime the number of shares that go into the calculation of the market-capitalization will be adjusted. This is done when the number of total issued shares is higher than the number of shares that trade in marketplace. It is called adjusting for the *free float*. The market-capitalization will then reflect the number of shares trading (free float) and not the total amount. This is often the case when large portions of the total issued stocks are held by single entities like the government (Bodie, Kane, & Marcus, 2008, p.45). It is important to know the weighting scheme of an index in order to know which movements the index reflects and which stocks that have the greatest effect on the return. When indexes, like the OBX, use market-capitalization weighting, the biggest firms will have the biggest impact on the performance of the index. As of 23rd of April 2010, there are three firms that make up over 45% of the total value of the OBX-index. Statoil alone make up 25% of the total value of the index. This means that movements in the Statoil price will affect the return of the OBX-index much more than Royal Caribbean Cruises which only makes up 2,35% of the total market value of the portfolio (NewsWeb, 2010).

Chapter 3: Investment Transaction Costs

According to Fabozzi (1998, p.338) investment transaction costs can be separated into *fixed* and *variable* costs:

$$\text{Transaction costs} = \text{Fixed costs} + \text{Variable costs}$$

3.1 Fixed transaction costs

The fixed components are *brokerage commissions*, *taxes* and *fees* that accrue when trades are carried out. They are assumed fixed even though such costs might differ between individual investors. This is because each investor knows the commissions, taxes and fees they face in the market. Brokerage commissions are paid to brokers for acting as middle men in the sale of securities. Profits made off of trades can be subject to taxation. Examples of fees are *clearing fees*, *custodial fees* and *transfer fees*. Clearing fees are charged by the clearing house for clearing derivatives. Clearing houses will charge the clearing member with this fee. The clearing members pass it on to the broker who further passes it on to the investor. Custodial fees are charged by institutions that hold investors' securities for *safekeeping*. Safekeeping means that institution holds investors securities, effectively reducing the risk of losing or having ones' securities stolen. Held securities are available for sale upon such request of the investor (Farlex Financial Dictionary, 2009). Investors are charged a *transfer fee* once a security is bought or sold as compensation for transfer of ownership. Fixed costs (Fabozzi, 1998, p.338):

$$\text{Fixed costs} = \text{Commission} + \text{fees} + \text{taxes}$$

The variable component of transaction costs are *execution costs* and *opportunity costs*:

3.2 Execution costs:

Execution costs are the difference between the execution price, i.e. the price at which investors trade, and the price that would have been observed had the trade not been carried out. The execution costs are separated into *market impact costs* and *market timing costs*.

$$\text{Execution costs} = \text{Market impact costs} + \text{Market timing costs}$$

3.2.1 Market Impact Costs

Market impact costs encompass the *bid-ask-spread* and any price movements attributable to the investor's trade. The market impact is also known as the *price impact* (Fabozzi, 1998, p.337). It refers to the impact buy or sell orders have on the price of a security. If the price movement is small then market impact costs will also be small. If price movements are big then market impact costs will also be considerable. The following section elaborates on the bid-ask spread, *liquidity* and *depth*, all linked with market impact costs.

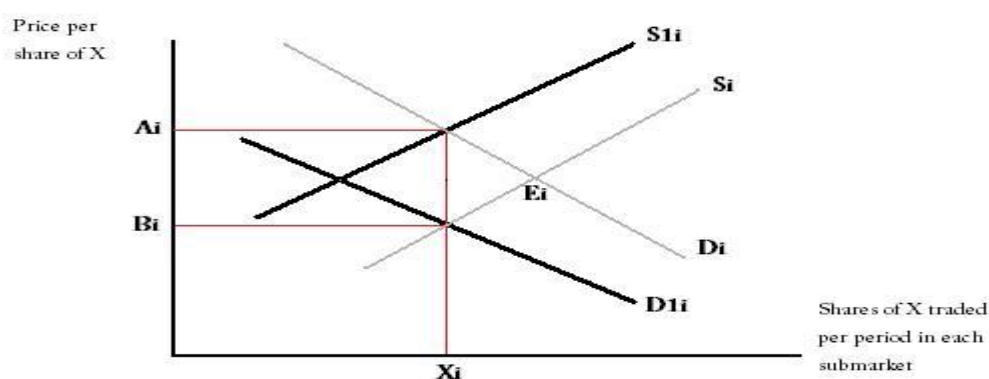
3.2.2 Bid-Ask-Spread

The bid-ask-spread is the difference between prices at which a *market-maker* will sell (*ask*) or buy (*bid*) a security. The quoted prices are for immediate sale or purchase. By quoting these prices a market is created, hence the name *market-maker*. The ask price is always higher than the bid price. This means that if an investor buys one stock at the ask price and immediately sells it back at the bid price, he will have a loss equal to the bid-ask-spread. Formulated differently; if the fair price of a stock is equal to the average of the spread, investors take a loss equal to half the spread when either buying at the ask price or selling at the bid price (Bodie, Kane, & Marcus, 2008, p.318).

In stock markets investor either place *market orders* or *limit orders* when they wish to buy or sell securities. When orders are completed they are said to be *filled*. Market orders are filled once a counter-offer to sell or buy is matched. The order is said to be filled at the current market price. Limit orders are orders to buy or sell a set amount of securities at a set price level or better. Investors placing a limit order to sell will sell the designated amount at a set

price or higher. Buyers will buy at set amount at a set price or lower. Market-makers exist because orders placed in the market are not always matched right away. They provide the service of *immediacy* by stating prices at which orders can be filled immediately.

Figure 3.1-The bid-ask spread



Source: Demsetz(1986) p.36

Figure 3.1 further elaborates. The vertical axis denotes the price per share of X . The horizontal axis gives the number of shares X , traded per period in each sub-market, denoted as X_i . S_i and D_i represents the supply and demand of security X of investors who wishes to have their orders filled immediately. The intersection between the two gives E_i . E_i is the average price at which X_i has and will be traded. However, investors cannot count on their orders being filled immediately. As such, S_{1i} and D_{1i} represents the market orders of supply and demand of security X_i at any time regardless of the number of counter-market orders present in the market. S_{1i} and D_{1i} represents the supply and demand of a market-maker who is willing to immediately fill counter-orders. At the intersection of S_{1i} and D_{1i} investors' market offers to buy is immediately filled at ask price A_i . At the intersection of D_{1i} and S_i investors' market offers to sell security X_i is immediately filled at bid price B_i . The bid-ask spread is given by the difference between A_i and B_i . The spread is set to compensate the market maker for the cost of providing liquidity (Demsetz, 1968,p.36). The next section elaborates.

3.2.3 Components of the bid-ask spread

The theories of the components of the bid-ask spread states that the width/size of the spread is determined by the costs and risk that the market-maker bears by providing liquidity. Huang & Stoll (1997) reviews the most important research on the bid-ask spread in their introduction. The three main components that make up the cost and risk of the market-maker are believed to be:

- Order cost
- Information costs
- Inventory handling costs

The *order cost* is the cost of transacting when both accounting for expenses and fees as well as labor and communications costs (Stoll, 1978,p.1144) . The source of *information costs* is information asymmetries regarding valuable information of a security. Consider the case when a small number of investors possess information regarding a security that indicates that the current market price does not reflect the security's real value. They are able to take positions in the security to profit from their private information. Their profit is matched by the loss of the market-maker from filling the investors' orders. The market-maker is assumed to not be able to distinguish between investors who have private information (*information traders*) and those who do not have private information (*liquidity traders*). Because the market-maker is not able to discriminate between information- and liquidity-traders he is forced to compensate by widening the bid-ask-spread. In practice this means that liquidity traders carry the information costs by having their order filled at a wider bid-ask-spread (Stoll,1978,p.1144).

Inventory handling costs relates to the fact that market-makers have to hold an inventory of the securities of which they trade. The two elements of the costs is *price risk* and *opportunity costs*.

Stoll (1978) models inventory holding costs on the assumption that the market-maker already has diversified portfolio in accordance with his level of risk aversion. By providing immediacy he takes on an inventory of securities that makes his total portfolio move out of line with his desired risk and return levels. This increases the price risk and reduces his total utility (p.1134). (Price risk is the risk that the price of the security might change in an unfavorable direction). To compensate for this the market-maker set the bid-ask-spread accordingly. The costs this represents for the investor is equal to the monetary compensation the market-maker demands for maintaining his utility level (p.1150). This means that acting as a market-maker induces opportunity costs that are compensate for by the size of the bid-ask-spread.

3.2.4 Liquidity and depth

Market impact costs are connected with *liquidity* and *depth*. Liquidity describes the speed and at what level of cost, an asset can be bought or sold at its current market price. If an asset is traded at its market price, quickly and at low costs it is said to be *liquid*. The opposite liquidity is *illiquidity*. For securities, illiquidity means that investors are not able to quickly sell or buy securities without having to pay a cost (Amihud, & Mendelson, 1991,p.56). Such costs are represented as premiums or discounts over the current market price. In relation to the bid-ask spread this means that illiquid securities will have a wider spreads than liquid securities. A wide spread can be justified on the market-maker's part to compensate for the level of price risk. Because an illiquid security is hard to sell without resorting to highly discounted prices, chances are that it will be part of the market maker's inventory portfolio for a longer time. This means in turn that chance of price movements increase as time passes (Fabozzi,1998,p326).

The depth is a measure of movements in the price of a security relative to the buy and sells orders placed on the security. If the price of a security has little movement for big orders then, that security is said to have a good depth. If the opposite is true, then that security is said to have a poor depth. A very poor depth will mean high market impact costs. Investors who trade in securities with poor depth will see unfavorable price changes as a result of their trading activities (Investopedia.com, 2010).

3.2.5 How the Bid-ask spread affects an investment

A theoretical approach¹ can be used to explain how the bid-ask spread affects the performance of an investment. Suppose an investor invests an amount S_0 in an asset S . The bid-ask spread is measured as percent of the ask price, presented in decimal form. The transaction costs of the investment is given by half the bid-ask spread, λ . The total transaction costs for the investor at time zero is $\lambda * S_0$. Because the investor pays half the bid-ask spread upon investment the total amount invested in the asset at time zero is $S_0 - S_0 * \lambda = S_0 * (1 - \lambda)$. Later, at time T the investors wishes to sell off his initial investment. The initial investment is now worth $S_T * (1 - \lambda)$. When the investor sells off his investment he will do so at the bid price which is λ less than the fair market price. The transaction costs from the sale are therefore equal to $S_T * (1 - \lambda) * \lambda$. The total amount received from the sale is given by $S_T * (1 - \lambda) - S_T * (1 - \lambda) * \lambda = S_T * (1 - \lambda)^2$. This means that the holding period return of the entire period can be written as:

$$r_S^* = \frac{S_T * (1 - \lambda)^2 - S_0}{S_0} \quad (3.1)$$

The return on the asset S for the period is given as r_S . Utilizing the return the value of the entire investment at time T can be rewritten $S_T = S_0 * (1 + r_S)$. Because $\lambda^2 \ll \lambda$, the part $(1 - \lambda)^2$ can be approximated to $(1 - 2\lambda)$. 2λ is the entire bid-ask spread. Taking the last paragraph into consideration, (3.1) can be expressed as:

$$r_S^* = r_S * (1 - 2\lambda) - 2\lambda \quad (3.2)$$

Because the bid-ask spread affects the return of the initial investment, it will also affect the expected return and standard deviation of the investment:

$$E[r_S^*] = E[r_S] * (1 - 2\lambda) - 2\lambda \quad \text{Expected return}$$

¹ Based on handout from supervisor Valeri Zakamouline: "Computation of M^2 Measure" 9th April 2010

$$Std[r_S^*] = \sigma_S * (1 - 2\lambda)$$

Standard deviation

3.2.6 Market timing costs

The market timing costs encompasses all other price movements at the time of a trade not attributable to the actions of the investor (Fabozzi, 1998, p.337).

3.2.7 Measuring Execution costs

The problem with measuring execution costs is that the true measure of the cost is not observable. Remember, the true measure is given as the difference between the execution price and the market price that would have been observed, had the investment not been carried out. A second problem is that execution prices depend on the supply and demand for the underlying security. This means that the execution price is influenced by the trading and the demand of liquidity of other investors (Collins, & Fabozzi, 1991, p.29).

However, one can try to estimate the market impact cost. A general way of measuring market impact costs is to take the difference between the execution price and benchmark for *the fair market price*:

$$Cost = Execution\ price - Fair\ market\ price$$

The fair market price is the price that would have prevailed in the market place had the investment not been executed (Collins, & Fabozzi, 1991, p.31).

The benchmarks for the fair market value can be divided into three different groups, *pretrade*, *post-trade* and *average-price*. Using pretrade benchmarks means benchmarking the execution price to the price level of a security that prevailed prior to the execution of an investment in that security. Examples of possible benchmarks are the last price at which the security traded, previous night's closing price and the average of the bid-ask spread. The justification of using

pretrade benchmarks is that the only way of knowing the effect a trade has on the price is to consider the conditions prior to executing the trade. Critics argue that a pretrade benchmark does not fulfill the requirement of being independent of the trade decision. This means that the investor can “game” the trade by structuring it in a way that apparently reduces the market impact costs (Collins, & Fabozzi, 1991,p.31)

Using post-trade benchmarks means benchmarking the execution price to benchmarks subsequent of a trade. Unlike pretrade benchmarks, post-trade benchmarks are not subject to gaming. Examples of post-trade benchmarks are any prices following the trade or the price following the closing price. The important thing is to choose a benchmark that is not observed too long after the initial trade, in order to account for the influence of a executed trade (Collins, & Fabozzi, 1991,p.31).

Average-price benchmarks establish a fair price benchmark, representative for a single trade day. Examples are the average of a security's high and low price or the trade-weighted average price (Fabozzi, 1998,p.342). The trade weighted average-price is calculated weighting each execution price by the number of shares bought at the price, relative to the total amount bought during the entire day of trading. The market impact cost is given as the difference between the trade weighted average-price and the buy price for each trade (Berkowitz, Logue, & Noser, 1988). Like the pretrade benchmarks these types of benchmarks are also subject to gaming. The trade weighted average price can be gamed by executing trades at the opening, closing and around large block trades. This means that investors following such a strategy will be reactive in their investing. They will exploit other investors demand for liquidity by investing at the same time as other investors execute large block orders. The results will never be better than mediocre. Average-price benchmarks are said to be put to better use as indicators of market timing costs. However, proponents of average-price benchmarks argue that these benchmarks are superior because unlike other benchmarks, they reflect an equilibrium price (Fabozzi, 1998,p.342).

3.3 Opportunity costs

Opportunity costs is measured as the difference between returns on desired investments and returns of the actual investment corrected for execution and fixed costs. It is the cost of not transacting, or not being able to transact in a desired manner. A simple way to think of it is instances where a portfolio manager is not able to acquire a certain stock with positive future returns (Fabozzi, 1998, p.338). The total return of the portfolio would have been greater in the future, had the investor been able to acquire the stock. The opportunity cost in this instance is the difference between the hypothetical portfolio returns including the stock, corrected for execution and fixed costs of buying the stock, and the actual portfolio returns.

$$\text{Opportunity costs} = \text{Desired returns} - \text{Actual returns} - \text{execution costs} - \text{Fixed costs}$$

3.3.1 Measuring Opportunity Costs

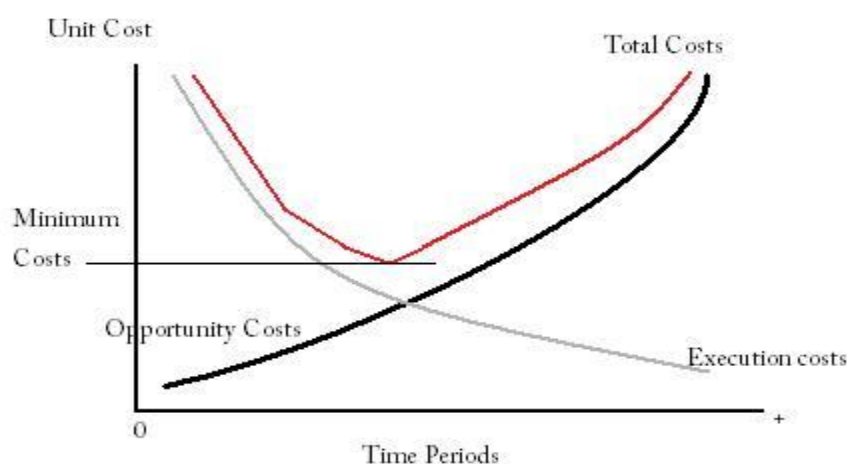
Obtaining a true measure of opportunity costs is practically impossible. Consider a stock investment strategy, with different investments in the same stock over a time horizon. The investor might observe perceived opportunities and decide to trade, only to fail (for whatever reason) to execute the desired trade in parts or as a whole. In order to measure the true opportunity costs one would have had to know what the performance of the stock return had been, had all the desired trades been executed at the desired times. However, by making the assumption that the observable performance of a security reflects the performance for which all investments are executable, one can measure an approximation of opportunity costs (Collins, & Fabozzi, 1991, p.29). Collins & Fabozzi (1991) measures opportunity costs by observing the performance of a portfolio that represents an investor's desired holdings. The opportunity costs are given as the difference in return between the desired portfolio and the actual portfolio, corrected for execution and fixed costs. They measure execution costs as the difference between the average bid-ask spread when the decision to invest was made and the average bid-ask spread when the trade was actually executed (p.32).

3.4 Transaction costs trade-off

Investors must consider all three cost elements. Changes in the fixed costs can influence execution costs and opportunity costs. Suppose commission rates are reduced. This changes

the risk-reward profile of the market-maker. To compensate the market-maker might widen the bid-ask spread, effectively increasing the execution costs of the investors. Execution costs can be reduced by postponing an investment until the market price is right. On the other hand, postponement of the investment can increase the opportunity costs, because delaying the investment means that it is possible to miss out on positive returns (Collins, & Fabozzi, 1991,p.29-30).

Figure 3.2-Cost Trade-offs, Execution vs. Opportunity Costs



Source: (Collins, & Fabozzi, 1991,p.30)

Figure 3.2 shows the trade-off between execution and opportunity costs. The vertical axis represents the cost per unit of security. The cost can be represented in currency or basis points. The horizontal axis gives the time periods running from 0. The time can be measured in minutes, hours or days. The execution cost is the falling gray line. It shows that execution costs are negatively correlated with time. The opportunity cost is represented by the increasing solid black line. It shows that opportunity costs are positively correlated with time. The red line illustrates the total costs, and “minimum costs” marks the best trade-off between execution and opportunity costs, for which the total costs are minimized. Figure 3.2 only shows the general shapes of the costs. The actual shapes will vary depending on the style of management (Collins, & Fabozzi, 1991,p.30).

Table 3.1 relates investment styles to execution and opportunity costs. Value motivated management styles like *Value* and *Growth* are concerned with the long term returns for increases in the price and earnings respectively (Fabozzi,1998,p.281). The long-term investment horizon means that such investors have time to delay their investments until they feel the price is right. This means low execution costs. It also means that the opportunity cost can be expected to be low. Suppose an investor plans on investing in a security and to hold it for no less than three years. If he postpones the execution of his investment for one day, the possible return on the security, corrected for execution and fixed costs, for that one day will be the opportunity cost. When considering the investment horizon such costs become insignificant.

Investors wishing to capitalize on private information are the aforementioned information-traders. *Earning surprise* is a term used for instances when the observed return on a security is out of line with its expected return, calculated by a security analysts (Fabozzi,1998,p.245). Information-trader is forced to make quick investments, because they do not know for how long the information will stay private. This means that the opportunity cost of not investing is high. It also means high execution costs: They will have their orders filled immediately by market-makers at the bid-ask-spread. If the depth of the stock is poor then large investments in the security in question will also drive the stock price in a unfavorably direction.

Index Funds are benchmarked to stock indexes. This means that the object of the fund is to replicate the returns of the fund. To do this investment managers follow a *passive* investment strategy by investing in the portfolios that replicate the index return. Such portfolios must often be rebalanced to keep up with the benchmark from day to day. This means that the time element is very important and that opportunity costs are very high. Large-cap(italization) stocks are often very liquid, thereby lowering the level of the execution costs. Small-cap stocks however, are often less liquid, meaning that execution costs can be expected to be higher.

Table 3.1-Management style versus Costs

Management Style	Trading Motivation	Liquidity Demands	Execution Costs	Opportunity Costs
Value	Value	Low	Low	Low
Growth	Value	Low	Low	Low
Earnings Surprise	Information	High	High	High
Index Fund Large-Cap	Passive	Variable	Variable	High
Index Fund Small-Cap	Passive	High	High	High

Source: Collins, & Fabozzi, 1991,p.30

3.5 Transaction costs and the bull fund

3.5.1 Fixed transaction costs of a bull fund

The daily re-balancing of the bull fund means that fixed transaction costs occur on a daily basis. We showed in section 2.3 how these costs are deducted from the total value of the fund and are equally distributed among all of the fund's issued shares. Both Handelsbanken and DnbNOR charge their funds with *variable depot costs* related to taking futures positions. Depot costs are paid to the *depot-receiver* (trans.). The depot-receiver has two main tasks: To keep and safeguard a fund's financial assets (futures contracts) and to make sure that the fund managers manage the fund in accordance with the statutes and agreements of the fund (Finanskomiteen, 2008). Both Handelsbanken and DnbNOR use in-house depot-receivers. The depot-receiver of Handelsbanken's Bull fund is *Svenska Handelsbanken AB*, branch office Norway. Handelsbanken also charge their fund NOK 100 per transaction in financial instruments in addition to variable depot costs. (Handelsbanken Kapitalforvaltning, 2008). The depot-receiver of DnbNOR's Bull fund is *DnB NOR Bank ASA*. The prospect of DnbNOR's Bull fund specifies that the depot cost is made up of three elements: NOK 2,5 in clearing fee per futures contract, NOK 400 per transaction to the depot-receiver and NOK 4 per transaction in *commercial papers* to Oslo VPS. (Commercial papers are unsecured short-term debt). The fund is in addition to this charged with commissions and banking fees stemming from the fund's investment in futures contracts (Dnbnor Kapitalforvaltning AS,2008).

3.5.2 Variable transaction costs and the Bull fund.

The Bull fund is affected by a bid-ask spread through the daily rebalancing. When the manager rebalances at the end of a trading day he must either go long more futures or close out long positions by taking short positions, and thus faces the bid-ask futures spread.

Suppose one long position taken at the beginning of a trading day. The futures price is F_0 which lies in the middle of the bid-ask spread of the market maker. The bid-ask spread is measured as percent of the ask price, presented in decimal form. Half a spread is λ and equal in both periods. This means that the ask price of the long futures is $F_0 * (1 + \lambda)$. When the futures position is closed out at time 1, it will be at $F_1 * (1 - \lambda)$, i.e. the bid price. The total long payoff is

$$payoff_{long} = F_1 * (1 - \lambda) - F_0 * (1 + \lambda)$$

Rewritten

$$payoff_{long} = (F_1 - F_0) - \lambda * (F_1 + F_0) \quad (3.3)$$

The payoff of short position at time zero that is closed out at time 1 is

$$payoff_{short} = (F_0 - F_1) - \lambda * (F_0 + F_1) \quad (3.4)$$

The second term on the right of both (3.1) and (3.2) are the costs of the bid-ask spread equal to half a spread of the prices at time zero and 1. Both equations show how the size of the bid-ask spread affects the payoff of single futures contracts.

Dnb NOR say that they rebalances their Bull fund at the futures market's close², with few exceptions, and adds that such exceptions are of little or no importance. Their futures orders are always filled by the market maker; Dnb NOR markets, which is obligated to quote bid and ask prices. Dnb NOR say that they do not consider the aspects of how and when their futures transactions might affect the futures price, i.e. market -impact and –movement costs. The exposure of the fund is in fact not twice the size of the fund's assets. This is because the regulation of the fund prohibits the fund manager to take derivative positions of more than 200% worth of the fund's total value. Dnb NOR says that the futures positions are about 1,95 times the total value of the fund. The level of exposure is to protect the fund from breaking the underlying regulations, which can happen if there are big market movements.

² Joakim Taaje Global Valuation, Risk & Performance Investment Operations Services, DnB NOR Kapitalforvaltning AS. E-mail 6th of May 2010

Chapter 4: Theory

4.1 The Theoretical Pricing Of Futures contracts

The theoretical pricing model of futures contracts is based off of the theoretical pricing model of *Forward Contracts*. Like futures contracts, forward contracts are contracts of trade of an underlying asset between two parties. The price is agreed upon today with delivery in the future. However, there are differences. Forward contracts trade in the *over-the-counter* market, usually between financial institutions and/or one of their clients. As such forward contracts are not subject to standardization. Contract specifications are drawn up and agreed upon by the trading parties themselves. This means for example that the maturity date of the contract will depend on the agreements of the two parties. Forward contracts are not marked-to-market and they usually go to delivery (Hull, 2008,p.39).

The Forward pricing models gives the theoretical prices in the absence of *arbitrage*. Arbitrage refers to trades that exploit price differences between markets for the same commodity.

Traders who take advantage of such opportunities are called *arbitrageurs*. They will buy a commodity in the marketplace with the lowest price and sell it in the market with the highest price. In financial economics arbitrage refer to trades that yield a positive profit with no risk and a “*zero-net investment strategy*” (Bodie, Kane, & Marcus, 2008, Glossary G-1). In the absence of arbitrage, prices are at such levels that arbitrage trade is not possible. Under such conditions it follows from the *law of one price* that “*assets (portfolios) with the same payoff must trade at the same price*³”.

The following section regards the theoretical pricing of futures contracts and is based off of Chapter 5 in Hull (2008) unless stated:

³ Valeri Zakamouline :Absence of arbitrage condition in pricing of securities. Financial Forward and Futures contracts. Lecture at UiA.

4.1.1 Pricing model of Forward Contracts

Assumptions: The following is true for some market participants:

- They are not subject to any transactions costs when they trade.
- They are subject to the same tax rate in all net trading profits.
- They can borrow money at the same risk-free rate of interest as they can lend money.
- They take advantage of arbitrage opportunities as they occur.

The market participants in question are assumed to be big derivatives dealers that actively partake in arbitrage trade to lock in risk-free profits. It is this activity that influences the relationship between forward and spot prices.

Notation for use in the model:

T = Time until delivery date in a forward or futures contract (in years)

S_0 = Price of the asset underlying the forward or futures contract today

F_0 = Forward or futures price today

r = Zero-coupon risk-free rate of interest per annum, expressed with continuous compounding, for an investment maturing at the delivery date.

There are two general expressions of the relationship between the forward and the spot price. One where the underlying asset pays no income and one where it does. If the underlying asset is a stock, dividends paid on that stock is an example of such income paid. The pricing model gives the forward price for which no arbitrage risk-free profits are possible.

No Income paying Investment Assets

It can be shown that the relationship between the current spot price of the underlying asset and the current forward price is given by. (4.1) gives the forward price, when the underlying pays no income:

$$F_0 = S_0 * e^{(r*T)} \quad (4.1)$$

The basis for this relationship is given by arbitrage trade. (4.1) holds when there is no possibility of arbitrage risk-free profits. The following examples illustrate:

Example: When $F_0 < S_0 * e^{(r*T)}$. Both the current spot and forward price is \$100, $r=0.05$ and $T=1$. Arbitrageurs will short sell the stock and receive \$ 100 which will be lent out at $r=0.05$. At the same time they take a long forward position to buy back the stock, priced at \$100 with delivery in one year. At maturity arbitrageurs will receive $(\$100 * e^{(0.05*1)})$ \$ 105.13 for the money lent out. They will use this to pay the forward price (\$100) and receive the stock which is turned back to the owner. The result is that they are left with a risk-free profit of $(\$105.13 - \$100)$ \$5.13.

Example: When $F_0 > S_0 * e^{(r*T)}$. Assume the same spot price, interest rate and time to maturity and a forward price equal to \$110. Arbitrageurs will now borrow \$100 at $r=0.05$ to buy the stock in the market place for \$100. At the same time they will take a short forward position to sell the stock in one year for \$110. At maturity the stock will be sold and the arbitrageurs will receive \$110. They will use this to pay off their loan, which at maturity stands at $(\$100 * e^{(0.05*1)})$ \$105.13. This means that they have earned a risk-free profit equal to $(\$110 - \$105.13)$ \$4.87.

Income paying Investment Assets

Income paid on the underlying asset has to be accounted for when deriving the forward price. As pointed out, such income can be dividends paid on a stock or coupons payments made on a

bond. Expression (4.2) takes such payments into account by adjusting (4.1) to include the present value of such payments at maturity. (4.2) gives the current forward price, for an underlying asset that pays known income, for which there are no arbitrage opportunities :

$$F_0 = (S_0 - I) * e^{(r*T)} \quad (4.2)$$

Where I is the present value of known income.

Note that I is the present value of *known income*. This makes (4.2) applicable when future dividends payments on a stock is know in both size and payment date. The following examples give scenarios of arbitrage trade when market prices are not in line with (4.2).

Example: Assume that the spot price of a stock is \$100. The stock pays a dividend of \$5 in 6 months ($T=0.5$), and both size and maturity is known to all participants. A forward contract with one year ($T=1$) is priced at \$105 and the one year risk-free rate is 5% ($r_1=0.05$) and the 6 month risk-free rate per annum is 3% ($r_2=0.03$). The present value of the dividends is thus $(\$5 * e^{(-0.03*0.5)}) = \4.93 . Arbitrageurs will buy the stock and take a short forward position. The stock purchase is financed with two loans. $(\$100 - \$4.93) = \$95.07$ is borrowed at $r_1=0.05$ with maturity in one year, while the rest $(\$100 - \$95.07) = \$4.93$ is borrowed at $r_2=0.03$ p.a with maturity in 6 months. In 6 months the arbitrageurs will receive \$5 in dividends. This will be used to pay off the 6 month loan which now stands at $(\$4.93 * e^{(r_2*0.5)}) = \5 . At maturity of the forward contract in one year arbitrageurs will make delivery of the stock and receive \$105. They will use some of this payoff to pay back their one year loan which now stands at $(\$95.07 * e^{(r_1*1)}) = \99.94 , thereby keeping the rest $(\$105 - \$99.94) = \$5.06$ as a risk-free profit.

Example: Assume that the spot price of a stock is \$100 and that the stock pays a \$5 dividend in 6 months ($T=0.5$). Maturity and size of the dividends is known to all market participants. The forward price of a contract with delivery, of the same stock, in 1 year ($T=1$) is \$90. The risk-free one year rate is 5% ($r_1=0.05$) and the risk-free 6 month rate is 3% p.a ($r_2=0.03$). Arbitrageurs will now short sell the stock and simultaneously take a long forward position to buy the stock back in one year. (Note that when shorting stocks, investors are obligated to return any dividends paid on the stock to the owner of the stock.) The proceedings from the

short sell will be invested in two parts. \$ 4.93 will be invested at $r_2=0.03$ for 6 months and the remaining $(\$100-\$4.93)$ \$95.07 will be invested at $r_1=0.05$ for one year. In 6 months the arbitrageurs will use the money invested at r_2 to compensate the stock owner for dividends paid. The \$4.93 invested at r_2 is now equal to the dividends payment of $(\$4.93 * e^{(r_2 * 0.5)})$ \$5. A time $T=1$, in one year, the investment at r_1 is now worth $(\$95.07 * e^{(r_1 * 1)})$ \$99.94. Arbitrageurs take delivery on the forward contract, they pay \$90 and return the stock to its original owner. This means that they will be left with a risk-free profit equal to $(\$99.94 - \$90)$ \$9.94.

Known income payments can also be expressed in terms of *yield*. When the known income is expressed in terms of yield, it means that the income is expressed as a certain percentage of the asset's value at time of income payment. If the yield is measured with continuous compounding (4.1) can be altered to include such an income yield:

$$F_0 = S_0 * e^{((r-q)*T)} \quad (4.3)$$

Where q is the continuously compounded annual income yield. In line with (4.2) such yield is deducted (here) from the risk-free return.

4.1.2 When are Forward prices equal to Futures prices?

Hull (2008) p.125-126, provides proof that forward and futures prices are equal when interest rates are assumed to be constant. The proof is reproduced in the following section. The proof is based on two strategies that both give the same payoff. Following the law of one price and the absence of arbitrage two identical strategies that both give the same payoff must be equally valued:

Scenario: Underlying asset is tradable in both forward and futures contracts. A futures contract matures in n days. Interest rate per day is assumed to be constant and given by: r . The futures price at the end of day i ($0 < i < n$) is F_i . The forward price at the end of day i

is G_i . At maturity the futures price will be equal to the spot price of the underlying (see: *The convergence property*).

Strategy 1: This strategy involves an investment in a risk-free bond and long futures positions. At the end of day 0 invest F_0 in a risk-free bond at r . At the same time take a long futures position of e^r contracts. At the end of day 1 that long position is increased to $e^{(r*2)}$ contracts, at the end of day 2 it is increased to $e^{(r*3)}$ contracts. The relationship is given as: At the end of day $i-1$ increase position to $e^{(r*i)}$. This continues until the end of day $n-1$.

The profits from the futures positions of day i is given by: $(F_i - F_{i-1}) * e^{(r*i)}$. All profits/losses are assumed to be reinvested at r with maturity at day n :

$$(F_i - F_{i-1}) * e^{(r*i)} * e^{r*(n-i)} = (F_i - F_{i-1}) * e^{(r*i)+(r*n)-(r*i)} = (F_i - F_{i-1}) * e^{(r*n)}$$

At the end of day n the total value of the futures strategy is given by:

$$[(F_n - F_{n-1}) + (F_{n-1} - F_{n-2}) + \dots + (F_1 - F_0)] * e^{(r*n)} = (F_n - F_0) * e^{(r*n)}$$

Following that the futures price will equal the spot price of the underlying asset at maturity (S_T), the total value of strategy 1 (including the bond investment) is:

$$F_0 * e^{(r*n)} + (S_T - F_0) * e^{(r*n)} = S_T * e^{(r*n)}$$

Strategy 2: This strategy involves investing in a risk-free bond and taking long forward positions. At the end of day 0 invest G_0 in a risk-free bond at r . At the same time take a long forward position of $e^{(n*r)}$ contracts at forward price G_0 . At maturity the payoff of the forward position combined with the bond can be presented as:

$$G_0 * e^{(r*n)} + (S_T - G_0) * e^{(r*n)} = S_T * e^{(r*n)}$$

As demonstrated, both strategies yield the same payoff. In the absence of arbitrage:

$$F_0 = G_0$$

4.1.3 The theoretical Stock index Futures Price

The relationship between the spot price of the futures prices is explained by the *cost of carry*. It is assumed that once a stock index futures contract is matched the short position will purchase the underlying portfolio and hold it until delivery. The purchase is financed with borrowed money at rate r , which represents a financial cost for the short position. However, any dividends (q) received while holding the underlying portfolio represents an income. The net sum of $(r - q)$ is called the *cost of carry*. The short position will demand compensation for such costs and the futures price is therefore adjusted (Collins, & Fabozzi, 1999, p.77). Assuming constant interest rates the current no-arbitrage theoretical price of a futures contract, on a stock index whose underlying portfolio pays a known dividends yield q and matures at time T , can be expressed as:

$$F_0 = S_0 * e^{((r-q)*T)} \quad (4.4)$$

4.1.4 Theoretical prices and the real world

The theoretical futures price is derived under assumption that cannot all be expected to hold in the real world. Collins & Fabozzi (1999, p.83) points to various reasons for why one single theoretical price might not be observed in the marketplace. First off, they assert that the *fair value* (theoretical) futures price is not observable as only one price but at a range of prices, with upper and lower bounds. These bounds set the arbitrage free perimeter and are the product of different costs (including transaction costs), differences between lending and borrowing rates and unstable cash flows. It is also important to point out that a fair price is not necessarily just one range of prices, but that each investor has his own fair price range. Each price range is said to be dependent on the investor's "*execution capability, ...cost structure, ...current portfolio position and ...cost of financing*" (Collins & Fabozzi, 1999, p.83). The cost of financing will vary between investors due to differing credit ratings.

4.2 How stock indexes move

The analysis in this paper is carried out by simulating movement paths for the price of a stock index. This is done under the assumption that the price of the stock index follows a continuous-time stochastic process. A continuous stochastic process is a random process followed by a variable, where changes in the value of the variable are uncertain and can happen at any time. The following section elaborates on the stochastic processes underlies the analysis in this paper.

4.2.1 Brownian motions

A Brownian motion is a continuous-time stochastic process, also known as known as *Wiener processes*. The change in a Brownian motion for any time period is normally distributed with a mean zero and a variance equal to the length of the time period. Different changes will also be independent of one another. This means that a change that happens in the future will not be influenced by the changes that came before it. Such an attribute mirrors a *weak-form market efficiency* of the stock market. When a market is weak form efficient, all stocks will be priced so that the price reflects all historical data of the stock. We will now show the properties of a Brownian motion:

Consider a variable Z that follows a Brownian motion. It will have these following properties:

For a change in Z over a small period of time Δt is given by:

$$\Delta Z = \varepsilon \sqrt{\Delta t} \quad (4.5)$$

ε is a randomly drawn number from a standard normally distribution: $\varepsilon \sim N(0,1)$. This means that the expected change, variance and standard deviation of ΔZ are:

$$E[\Delta Z] = E[\varepsilon \sqrt{\Delta t}] = 0 * \sqrt{\Delta t} = 0 \quad \text{Expected change}$$

$$Var[\Delta Z] = Var[\varepsilon \sqrt{\Delta t}] = \Delta t * Var[\varepsilon] = \Delta t \quad \text{Variance}$$

$$Std[\Delta Z] = \sqrt{Var[\Delta Z]} = \sqrt{\Delta t} \quad \text{Standard Deviation}$$

The changes in Z are assumed to be independent of one another. It can therefore be shown that the variance is over a period of time is equal to the length of the period, and that the

expected change of the period is zero. Consider a time period T consisting of N small equally sized time periods Δt :

$$N = \frac{T}{\Delta t} \quad (4.6)$$

The total change in value of Z over the period can be expressed as:

$$[Z_T - Z_0] = \sum_i^N \varepsilon_i \sqrt{\Delta t} .$$

The expected change, variance and standard deviation is:

$$E[Z_T - Z_0] = E\left[\sum_i^N \varepsilon_i \sqrt{\Delta t}\right] = \sum_i^N (E[\varepsilon_1] + E[\varepsilon_2] + \dots + E[\varepsilon_N]) = 0 \quad \text{Expected Change}$$

$$Var[Z_T - Z_0] = Var\left[\sum_i^N \varepsilon_i \sqrt{\Delta t}\right] = \Delta t * (Var[\varepsilon_1] + \dots + Var[\varepsilon_N]) = N * \Delta t = T \quad \text{Variance}$$

$$Std[Var[Z_T - Z_0]] = \sqrt{Var[Z_T - Z_0]} = \sqrt{T} \quad \text{Standard Deviation}$$

(Hull, 2002, p.217-219)

4.2.2 Geometric Brownian motion

A geometric Brownian motion is a variable whose changes over time are given by:

$$\frac{dS}{S} = \mu dt + \sigma dZ \quad (4.7)$$

dS is the change in variable S . μ and σ are constants and dZ is a Brownian motion. For a small change in time, dt , the expected rate of change in S is given by μdt . μ is the *drift rate*. The drift rate is the average increase in value per time unit in a stochastic process. The variance of the rate of change is given by $\sigma^2 dt$. σ is the volatility of the process. The solution to (4.7) is:

$$S_T = S_0 * e^{((\mu - 0.5\sigma^2)*T + \sigma * Z_T)} \quad (4.8)$$

By using (4.5) we re-write (4.8) as:

$$S_T = S_0 * e^{((\mu - 0.5\sigma^2)*T + \sigma * \sqrt{T} * \varepsilon)} \quad (4.9)$$

Where ε still is a random drawing from a standard normal distribution. (4.9) is the equation we use to simulate stock index movements. S_T and S_0 are the stock index prices at time T and 0. μ and σ are the expected continuously compounded [cc] return and standard deviation. The exponentiated part of (4.9) calculates the return of the stock index for the period. A geometric Brownian motion is preferred over a standard Brownian motion because stock index prices have a log-normal distribution under a geometric Brownian motion. The log-normality means that the price of the stock index never can be less than zero (Back, 2005).

4.2.3 Log-normally distributed prices

A variable has a log-normal distribution if the natural logarithm of the same variable is normally distributed. In the instance of stock indexes this means that the index prices will be log-normally distributed if the cc return is normally distributed:

$$R_{(0,T)} = \ln\left(\frac{S_T}{S_0}\right) \quad (4.10)$$

$R_{(0,T)}$ is the normally distributed cc return over the period 0 to time T . It is equal to the exponentiated part of (4.9). By taking the exponential of both sides:

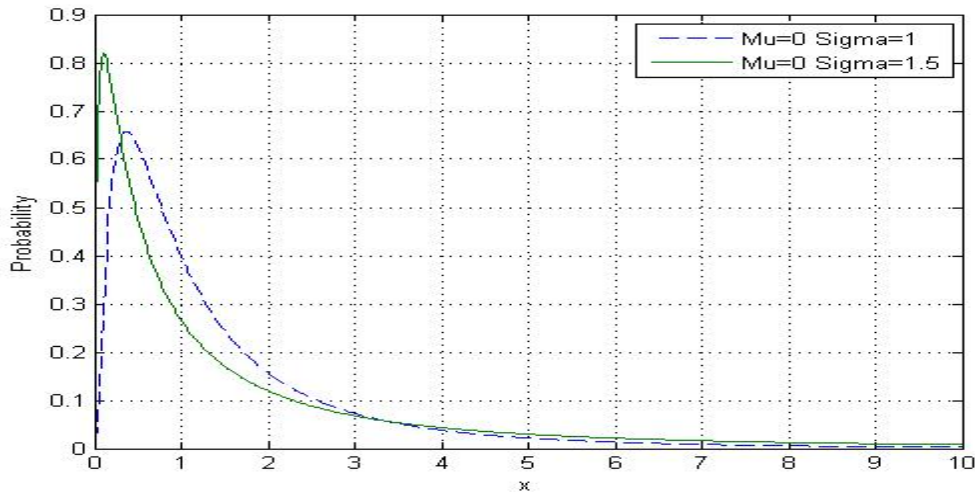
$$e^{R_{(0,T)}} = \frac{S_T}{S_0}$$

This can be re-written as

$$S_T = S_0 * e^{R_{(0,T)}} \quad (4.11)$$

Thus S_T can never be negative because of the exponentiation of the cc return. The typical shapes of log-normally distributions are given in figure 4.1:

Figure 4.1-Log-normal distributions



The log-normal density function **Source:** Matworks.com (2010)

Figure 4.1 shows the log-normal distribution for two associated normal distributions, $N(0,1)$ and $N(0,1.5)$, both with a mean equal to 0, and standard deviations = 1 and 1.5. The graphs are constructed based on MATLAB scripts from Matworks.com (2010). Both these log-normal distributions are for positive values of X , and both are skewed to the right. The latter is also a feature of log-normal distribution. For stock prices this means that extremely high prices far away from the mean and are unlikely to observe. The single “humps” in the log-normal distributions, means that the distributions are *unimodal*. They occur to the left of the mean (McDonald, 2006, p.593-594).

4.3 Performance measures

The following section describes the performance measures, *Sharpe ratio* and the *M-squared measure*, which are used, to evaluate the performance, in the analysis part of this paper.

4.3.1 The Sharpe ratio

The Sharpe ratio is a popular performance measure often used to evaluate and rank the performance of investment managers. The Sharpe ratio measures the trade-off between expected excess return and the risk of the excess return, on a risky investment. The risk is measured as the standard deviation of the excess returns. The excess return is the difference between the expected return on a risky investment and the risk-free expected return on a risk-free investment. It is also known as the *risk premium*. (4.12) gives the Sharpe ratio in its general form (Bodie, Kane, & Marcus, 2008,p.139):

$$\text{Sharpe ratio} = \frac{\text{Risk premium}}{\text{Standard deviation of risk premium}} \quad (4.12)$$

4.3.2 Deriving the Sharpe ratio

Suppose an investor has chosen the composition of a risky portfolio that he wants to invest in. The next step for him is to decide the portion of his investment budget, y , that he wants to invest in the risky portfolio, p , and the portion $(1 - y)$ that he wants to invest in a risk-free asset. $y + (1 - y) = 1$. The return on the risky portfolio is denoted r_p , the expected return is denoted $E[r_p]$ and the standard deviation (risk) of returns is denoted σ_p . The return on the risk-free asset is denoted r_f . Because it is risk-free the standard deviation of the risk-free asset is equal to zero.

$$r_c = y * r_p + (1 - y) * r_f \quad (4.13)$$

(4.13) give the return of the *complete portfolio*, denoted r_c . The complete portfolio is comprised of y parts of the risky portfolio and $(1 - y)$ parts of the risk-free asset. Taking the expected return on the complete portfolio gives:

$$E[r_c] = y * E[r_p] + (1 - y) * r_f$$

Rewriting it:

$$E[r_c] = r_f + y * (E[r_p] - r_f) \quad (4.14)$$

(4.14) give the expected return on the complete portfolio. r_f is the base rate of return. The section $y * (E[r_p] - r_f)$ is the expected *risk premium*, depending on the portion y invested in the risky portfolio and the expected risk premium, $(E[r_p] - r_f)$, of the risky portfolio. It is the expected excess return over the expected risk-free return. Because investors are risk averse they demand a positive risk premium in order to invest in a portfolio.

$$\sigma_c = \sigma_p * y \quad (4.15)$$

(4.14) give the standard deviation of the complete portfolio. It is equal to the standard deviation of the risky portfolio times the portion y invested in the risky portfolio. This makes the standard deviation of the complete portfolio proportional to the portion y invested in the risky portfolio. By including a risk-free asset in the complete portfolio total standard deviation is thereby reduced.

Figure 4.2-The investment opportunity set with a risky asset and a risk-free

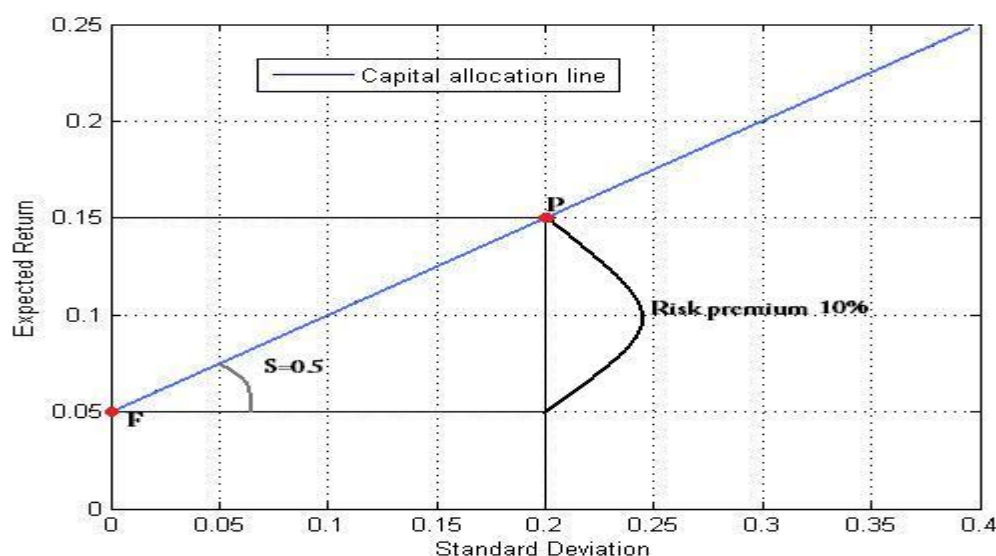


Figure 4.2 plots the characteristics of the complete portfolio in an expected return/standard deviation plane, for different values of y . The risky portfolio, marked **P**, has an expected return, $E[r_p] = 15\%$ and a standard deviation $\sigma_p = 20\%$. The risk-free asset, marked **F**, has an expected return, $r_f = 5\%$. The **Risk premium** is given by: $E[r_p] - r_f = 10\%$. If the investor decides to invest all his funds in the risky portfolio, then $y=1$. Such a complete portfolio will have the same expected return and standard deviation as the risky portfolio. If the investor decides to invest all his funds in the risk-free asset then $y=0$, and the complete portfolio will have an expected return equal to r_f and zero standard deviation. For values of y between 1 and 0, the complete portfolio will lie on the blue line between **F** and **P**. The blue line is known as the *Capital Allocation line*. As y increases from zero toward one, the complete portfolio will move from **F** to **P**. This means that bigger and bigger portions will be invested in the risky portfolio. Following (4.14) and (4.15), both the expected return and standard deviation of the complete portfolio will also increase for higher values of y . The slope of the Capital allocation line is marked in figure 4.2 as **S**. It is derived by first rearranging (4.15):

$$y = \frac{\sigma_c}{\sigma_p} \quad (4.16)$$

Substituting it into (4.14):

$$E[r_c] = r_f + \frac{\sigma_c}{\sigma_p} * (E[r_p] - r_f)$$

Rearranging:

$$E[r_c] = r_f + \frac{(E[r_p] - r_f)}{\sigma_p} * \sigma_c \quad (4.17)$$

(4.17) is the expected return of the complete portfolio. r_f is the intercept and $(E[r_p] - r_f)/\sigma_p$ is the slope of the capital allocation line. The slope is actually the Sharpe ratio:

$$S = \frac{(E[r_p] - r_f)}{\sigma_p} \quad (4.18)$$

The slope gives the excess return to standard deviation, and is therefore called the *reward-to-variability-ratio*, or the *Sharpe ratio*. The Capital allocation line gives the expected return of the complete portfolio as a function of r_f plus its standard deviation times the Sharpe ratio of the risky portfolio. The Capital allocation line gives all the combinations of expected return and risk available to all investors. The presentation of the capital allocation line in the expected return/standard deviation plane is called the *investment opportunity set*. (Bodie, Kane, & Marcus, 2008, p.177-179).

It must be noted that the Sharpe ratio will vary systematically with the time horizon of the investment. This is because the continuously compounded return grows at a constant rate proportional to time while the standard deviation grows slower, at the square root of time. This means that the Sharpe ratio grows for longer investment horizons, at the rate of the square root of time (Bodie, Kane, & Marcus, 2008, p.154).

4.3.3 The M-squared Measure of Performance

When ranking investment performance by the Sharpe ratio, we rank portfolios by their excess return relative to the risk of the excess return. Consider two risky portfolios, **A** and **B**. The Sharpe ratio of portfolio **A**, S_A , is 0.8 and the Sharpe ratio of portfolio **B**, S_B is 0.7. This implies that portfolio **A** has a better reward-to-volatility than portfolio **B**. However, what does the (0.8-0.7) 0.1 difference in Sharpe ratio imply? What is the economical interpretation of this difference? The M-squared measure of performance sheds light on this difference by comparing the returns of an *adjusted portfolio* with the benchmark portfolio. The adjusted portfolio is a combination of a risky portfolio and a risk-free asset, which gives the adjusted portfolio a standard deviation equal to the standard deviation of the benchmark portfolio.

Consider portfolios **A** and **B** once more. Suppose that portfolio **A** is a market index and that portfolio **B** is a risky portfolio that is benchmarked to **A**. The expected return and standard deviation of portfolio **A** is $E[r_A]$ and σ_A . The average return and standard deviation of risky portfolio **B** is $E[r_B]$ and σ_B . The expected return on a risk-free asset is denoted r_f . The portion of the risky portfolio in the adjusted portfolio **C** is given by:

$$a = \frac{\sigma_A}{\sigma_B} \quad (4.19)$$

Meaning that the portion invested in the risk-free asset is:

$$(1 - a) \quad (4.20)$$

The Expected return on the adjusted portfolio **C** is

$$E[r_C] = (1 - a) * r_f + a * E[r_B]$$

Or

$$E[r_C] = SR_B * \sigma_A + r_f \quad (4.21)$$

Where SR_B is the Sharpe ratio of risky portfolio B

The M-squared measure of performance is given as:

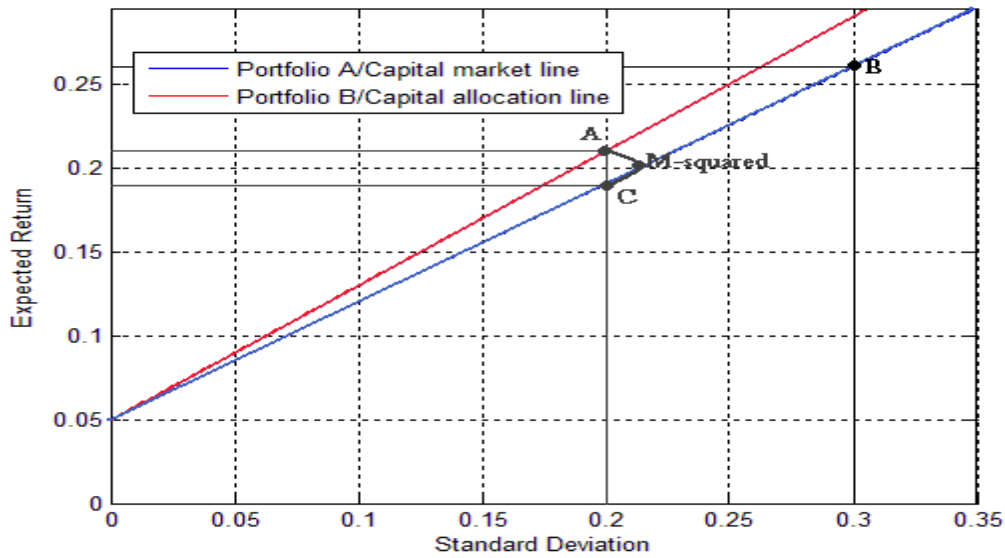
$$M^2 = E[r_C] - E[r_A] \quad (4.22)$$

By substituting (4.21) for the expected return of **C** and rewriting, M^2 is expressed as:

$$M^2 = (SR_C - SR_A) * \sigma_A \quad (4.23)$$

SR_A is the Sharpe ratio of benchmark market index **A**.

Figure 4.3-The M-squared measure of portfolio B



Source: Bodie, Kane, & Marcus(2008) page.856

Figure 4.3 shows portfolio **A** and **B** plotted in an expected return/standard deviation plane.

The risk-free return, $r_f = 5\%$. Portfolio **A** has an Expected return, $E[r_A] = 21\%$ and a standard deviation, $\sigma_A = 20\%$. The Sharpe ratio of portfolio **A**, following (4.18)

: $SR_A = (21\% - 5\%)/20\% = 0.8$. The red capital allocation line of portfolio **A** is referred to

as the *Capital Market line*, because portfolio **A** is the market index. Portfolio **B** has an expected return, $E[r_B] = 26\%$ and a standard deviation, $\sigma_B = 30\%$. The Sharpe ratio of portfolio **B**, following (4.18): $SR_B = (26\% - 5\%)/30\% = 0.7$. The blue line in figure 4.3 is the capital allocation line of portfolio **B**. The capital market line of portfolio **A** lies above the portfolio **B**'s capital allocation line, because portfolio **A** has a higher slope/Sharpe ratio relative to portfolio **B**. Point **C** in figure 4.3 marks the expected return and standard deviation of the adjusted portfolio **C**. Following (4.19) and (4.20), portfolio **C** is composed of $a =$

$(20\%/30\%) = 2/3$ of the risky portfolio **B** and $(1 - a) = (1 - 2/3) = 1/3$ part risk-free asset. The standard deviation is thus equal to that of portfolio **A**, 20%. The expected return, $E[r_c]$ of adjusted portfolio **C** is computed using (4.21): $(0.7 * 20\%) + 5\% = 19\%$. The **M-squared measure** of portfolio **B** is thereby given, utilizing (4.22): $(19\% - 21\%) = -2\%$. This is

marked in figure 4.3 as the horizontal gap between **A** and **C** (Bodie, Kane, & Marcus, 2008,p.855-856).

4.3.4 Relating the performance measures to the Bull funds.

The Sharpe ratio of a Bull fund can be shown to be equal to the Sharpe ratio of the underlying stock index. Suppose a stock index has an expected return, $E[r_s]$ and a standard deviation, σ_s . The risk-free rate is r_f . This means that the Sharpe ratio of the stock index is:

$$SR_s = \frac{E[r_s] - r_f}{\sigma_s}$$

Suppose there is a Bull fund that benchmarks to the stock index. Suppose that the fund is designed to yield y the expected return of the stock index at y the standard deviation of the stock index. The expected return of the Bull fund is thereby given by (4.14):

$$E[r_B] = r_f + y * (E[r_s] - r_f). \text{ The standard deviation is given by (4.15): } \sigma_B = y * \sigma_s.$$

The Sharpe ratio of the Bull fund can thereby be written as:

$$SR_B = \frac{E[r_B] - r_f}{\sigma_B}$$

Substitute the full terms of $E[r_B]$ and σ_B into SR_B :

$$SR_B = \frac{(r_f + y * (E[r_s] - r_f)) - r_f}{y * \sigma_s}$$

The r_f s and y s zeros out:

$$SR_B = \frac{E[r_s] - r_f}{\sigma_s} = SR_s$$

This means that the theoretical M^2 measure of the Bull fund is:

$$M_B^2 = (SR_B - SR_s) * \sigma_s$$

$$M_B^2 = (SR_s - SR_s) * \sigma_s$$

$$M_B^2 = 0$$

In theory the Sharpe ratio of a Bull fund and the underlying stock index should be identical which in turn means that theoretically, the M-squared measure should be equal to zero.

Chapter 5: Other written work on LETFs

This chapter presents studies and analysis regarding LETFs. Although LETFs are a fairly new financial creation, there is a fair amount of working papers and research notes written on the subject. A lot of the topics concern the LETFs' ability to yield double returns, how returns are over longer periods of time and what factors that affects the realized return.

Co (2009) models the value of a LETF and shows how the value is negatively affected by the volatility of the underlying index. Co compares the variance related loss to the gamma loss of options. The author shows how the variance causes losses with an example of a perfectly "hedged" portfolio of a long and short LETF. The portfolio suffers losses over time. The size of the losses is positively correlated with the volatility of the underlying benchmark.

Hill & Foster (2009) studies the effect of compounding returns of LETFs and the daily return for different periods of time, ranging from two days to six months. The authors find that there is a high probability that LETFs can deliver double daily returns over longer periods of time. The probability is said to be greater for shorter periods of time and low values of index volatility.

Haga and Lindset (2009) analyze the Bull and Bear funds offered by Handelsbanken and DnbNOR. The paper analyzes the performance of the funds using both simulations and empirical data. Haga and Lindset show how higher values the risk-free rate negatively affects the returns of the funds, and that the effects become greater over time. They also run regressions on the empirical data, and analyze if the funds indeed provide double and minus double returns relative to the benchmark. They find that both providers' funds have failed to give an exact double or inverse double return, but that both providers have come close, with Handelsbanken funds coming closest. The paper points out that the futures exposure of the funds are big enough to achieve the double exposure and speculates that this might be due to the transaction costs involved with rebalancing the funds.

Lu, Wang, & Zhang (2009) study the long term performance of both long and short leveraged ETF. Based on empirical data, of LETFs offered by Proshare, Lu, Wang and Zhang study the performance of LETFs for different time horizons. They find that both long and short leveraged ETFs can be expected to provide the investor with twice the return of the benchmark for investment periods no greater than one month. The analysis reveals that for longer holding periods the short LETFs returns diverge significantly from its objective of twice the inverse return of the benchmark for periods equal to a quarter of a year or greater. The long LETF returns show a divergence from double returns for a yearlong investment period. The article concludes that the returns of the LETFs are negatively affected by the quadratic variation and auto variation over a holding period, the latter having the biggest impact. Both quadratic variation and auto variance are products of the length of the time period and variance.

Avellaneda & Zhang (2009) presents a pricing model for LETFs. The pricing model connects the value of a LETF to the value of an underlying index or corresponding ETF. The model links the return of the LETF to the return of the underlying and the realized variance of the underlying. The paper argues that a LETF return is mainly affected by realized variance of the underlying. Avellaneda & Zhang validates the model by testing it against empirical data of a total of 56 funds. They also propose a dynamic hedging strategy involving LETFs for replicating the returns of the underlying index or ETF, and conclude that such a strategy is foremost reserved for active traders.

Chapter 6: Research method

In Chapter 7 we analyze the performance of a bull fund with and without transaction costs, and also try to determine any impacts the transaction costs may have on the performance. This is done by simulating the performance of an underlying stock index, a bull fund without transaction costs and a bull fund with transaction costs, using a MATLAB script. The analysis is a simplification and transaction costs are represented only by the bid-ask spread. (See chapter 3.5 for a overview of all transaction costs related with the rebalancing of the Bull fund.) The bull fund with transaction costs is called **V** and bull fund without transaction costs is called **U**. The index is named **S**. The performance of the two funds are benchmarked to **S** and measured using the M-squared measure, which is calculated by the script. The script is run for different time horizons, and values of annual expected return (μ) and annual standard deviation (σ) of **S**. The performance analysis is carried out for different values of annual expected return and annual standard deviation of **S**, in order to determine the impact of such values on the performance of both **U** and **V**. The values of **U** and **V** are then compared in order to analyze how transaction costs might affect the bull fund's performance under different values of expected return and standard deviation. All M-squared measures for every run is stored in tables which are organized after the time horizon, expected return and standard deviations of **S**, for which the script was run. There are three tables in total: Table 7.1 contains the M-squared measures of **U**, named **M2U**. Table 7.2 contains the M-squared measures for **V**, named **M2V**. Table 7.3 contains the difference between **M2U** and **M2V**, named **M2D**.

An analysis of the expected return and standard deviation of **U** and **V** is also carried out to see if it is possible to explain some of the results from the M-squared analysis. Expected returns standard deviations are estimated for **U** and **V** for daily, weekly and monthly time horizons and different values of μ and σ . The estimated expected returns are calculated with a 95% confidence interval [CI] and are assumed to have a student t-distribution. The estimated expected returns with 95% CIs of **U** and **V** are compared to see if there are significant differences. The standard deviations are also estimated with 95% CIs and compared. The estimated standard deviations are assumed to have a chi-squared distribution. In comparison of both estimated expected returns and standard deviations, the criteria for significant

differences is that the 95% CIs do not overlap. The results from simulations are stored in tables and referenced in the analysis section.

In addition, simulations are also run to analyze the effect of the risk-free rate on expected returns, the effect of the management fee on the expected returns as well as a comparison of the estimated standards deviations of **U** and **V** to the estimated double standard deviation of **S**. All results are stored in tables which are referenced in the respective analysis sections.

6.1 Assumptions about the stock index, futures contracts and the bull fund

The Stock index is assumed to follow a geometric Brownian motion. The index price S_T , at time T is given by (4.9). The cc return, R , of the stock index over a small time period, Δt , is given by the exponentiated expression of (4.9): $((\mu - 0.5\sigma^2) * \Delta t + \sigma * \sqrt{\Delta t} * \varepsilon)$. The index price at time 1 can therefore be expressed as:

$$S_1 = S_0 * e^{R_1} \quad (6.1)$$

The futures contracts are priced according to (4.4). It is assumed that there is no dividend paid, so $q = 0$. The futures price of a contract at time zero, with delivery at time **T** is therefore given as:

$$F_{(0,T)} = S_0 * e^{r*T} \quad (6.3)$$

The futures price for the next day, day 1, is given as:

$$F_{(1,T)} = S_1 * e^{r*(T-\Delta t)} \quad (6.4)$$

By substituting S_1 with (6.1) and rewriting $S_0 = F_{(0,T)}/e^{r*T}$, (6.4) becomes:

$$F_{(1,T)} = F_{(0,T)} * e^{(R_1 - r*\Delta t)} \quad (6.5)$$

6.1.1 The model of a Bull fund without costs

The value of **U** at day 1 is given as:

$$U_1 = U_0 * e^{r*\Delta t} + 2 * U_0 * \frac{F_{(1,T)} - F_{(0,T)}}{F_{(0,T)}} \quad (6.6)$$

The first part on the right side considers the fact that the fund is made up of cash and that a risk-free interest rate is earned daily on the cash holdings. The second part on the right side reflects the double daily returns of the futures contracts. The returns on the futures contracts are multiplied by 2, which is in turn multiplied by the value of the fund at the beginning of the period. Substituting (6.5) into (6.6) gives:

$$U_1 = U_0 * (e^{r*\Delta t} + 2 * (e^{R_1 - r*\Delta t} - 1)) \quad (6.7)$$

The expected return of U at time n can be expressed:

$$E[U_n] = U_0 * (1 + (2\mu - r)\Delta t)^n$$

The derivation of the expected return is given in the appendix. The model for U and the derivation of both the model and the expected return is based on Haga & Lindset(2009). What is important to notice (,what Haga and Lindset show in their paper,) is how the risk-free return affects the expected return of the Bull fund U . As long as the risk-free rate is positive, the expected return of the fund cannot be expected to be twice the size of the expected return of the benchmark. The effect of a positive interest rate also becomes greater for longer time periods.

6.1.2 The model of a Bull fund with costs

The model of V builds on the U model with two additional assumptions: First the fund is charged daily with a management fee. Second, the fund is rebalanced daily and is therefore charged with transaction costs, represented by the bid-ask spread of futures contracts. The value of the fund after the management fee has been deducted, but before rebalancing is:

$$V_1^* = V_0 * \left(e^{r*\Delta t} + 2 * (e^{R_1 - r*\Delta t} - 1) \right) * (1 - (e^{f*\Delta t} - 1)) \quad (6.8)$$

The management fee is represented by f , in the last part of the right side. The management fee is assumed to be cc every day, before rebalancing. After the management fee has been deducted the fund manager needs to rebalance the number of futures contract, so that the holdings are twice the size of the value of the fund:

$$N_1 = \frac{2*V_1^*}{F_{(1,T)}} \quad (6.9)$$

N_1 is the number of futures contracts needed for a double exposure.

$$dN = abs(N_{t+1} - N_t) \quad (6.10)$$

(6.10) give the change in total amount of futures contracts after rebalancing. The number is absolute. N_t is the number of futures contracts at the beginning of the day t .

$$V_1 = V_1^* - (dN * spread * F_{(1,T)})/2 \quad (6.11)$$

(6.11) is the value of the fund after both deducting the management and half the bid-ask-spread. The value of the bid-ask-spread is calculated by the second term on the right side.

6.2 The parameter values

The MATLAB script is run for different values of annual expected return and annual standard deviation of S , for different time horizons. The annual expected return is referred to as μ and the annual standard deviation is referred to as σ . Table 6.1 contains all the values of μ and σ for which the simulation is run. The simulation is run for every pair of μ and σ .

Table 6.1–Values of μ and σ

μ	10%	15%	20%	30%
σ	20%	25%	30%	40%

The time periods considered are *daily*, *weekly*, *monthly*, *quarterly*, *semi-annual* and *annual*. Simulations are run for each of these time horizon, for every pair of μ and σ . Note that it is only the M-squared measures that are estimated for all these time horizons. The time horizon is denoted by T . T measures time in units of years. The number of steps per time horizon is denoted n . The simulation assumes that the year consists of 250 trading days. This means that the number of steps per time horizon is given by $250 * T = n$. Table 6.2 gives the values of T and the number of steps for each time horizon.

Table 6.2-Values of T and number of steps for each time horizon

	Daily	Weekly	Monthly	Quarterly	Semi-annual	Annual
T	0,004	0,02	0,08	0,24	0,5	1
n	1	5	20	60	125	250

The value of the whole bid-ask spread is set to 0,316%. The number is derived from OBX-futures spread data available on OSE homepage www.oslobors.no. The data is for the futures contract *OBX0D* which had delivery date in April of 2010. The spread data has 120 observations bid and ask prices running from 12th of October 2009 to the 9th of April 2010. The spread of each observation is calculated as the average price of the bid and ask prices divided by the ask price. The designated spread value is the average of all these calculated spreads.

Because this paper's objective is to determine the effect of the transaction costs, the management fee is set to 0%. However, in section 7.6 an analysis of the effect of management fee is carried out. The management fee f , is then set to 0,8% per annum. At their inception, both Bull funds from Handelsbanken (Handelsbanken kapitalforvaltning, 2008) and DnbNOR Dnbnor (Kapitalforvaltning AS, 2008), had a management fee equal to 0,8% per annum.

Other values include the start value of **S**, **U** and **V** and the risk-free rate. The start value of all three is set to 100. The risk free rate, **r** is set to 5% p.a. with the exception of simulations runs for the analysis in section 7.7 where the risk-free rate is also set to **r**= 0%, and weekly estimated expected returns for both values of **r** are compared to determine the effect of the risk-free rate.

6.3 Running the MATLAB script

The MATLAB script, used to compute **M2U**, **M2V** and **M2D**, and estimate expected returns and standard deviations is given in its entirety in the appendix. The part of the script is constructed to simulate the price/value paths of **S**, **U** and **V**, over a time horizon, with a given number of steps **n**. One simulation simulates one path for each of **S**, **U** and **V**. The number of paths simulated is set to **m**. Longer time horizons means more steps, which in turn commands more computer power in order to simulate. Because of this, the number of simulated paths differs between some of the time horizons. Daily simulations were run 7 million times. Weekly simulations were run 3 million times. For monthly measures the simulation was run 0,9 million times. Quarterly simulation were run 0,3 million time, semi-annual were run 0,1 million times and annual simulations were run 0,08 million times. The computer on which the script was run was “stress-tested” for different amounts **m** to find the highest possible number of simulations that was possible to run given the computer power.

Script 6.1-The code used to run simulations of paths

```
for j=1:m
    for i=1:n
        R=( (mu-0.5.*sigma.^2).*dt+sigma.*sqrt(dt).*randn); %Stock index return per period dt
        S(j,i+1)=S(j,i).*exp(R); %Price path of stock index
        F(j,i+1)=S(j,i+1).*exp(r.*(T-i.*dt)); % Futures price path
        Vt = V(j,i)*( exp(r*dt)+M*(exp(R-r*dt)-1) )*(1-(exp(f*dt)-1)); %Value of V
                                                %less management fee

        N(j,i+1) = M*Vt/F(j,i+1); %Number of contracts per period
        dN = abs( N(j,i+1)-N(j,i) ); %Change in number of contracts
        U(j,i+1) = U(j,i)*(exp(r*dt)+M*(exp(R-r*dt)-1)); %Value of U per period
        V(j,i+1) = Vt-dN*spread*F(j,i+1)/2; %Value of V less spread and
                                                %management fee per period
    end
end
```

Script 6.1 shows the excerpt of the MATLAB script that is used to simulate one path for each of **S**, **U** and **V**. The description of each line is given by the text at the right side in green color. They are the presented models in chapter 6.1. After the simulations have been run once, the

script calculates the period return of **S**, **U** and **V**, and stores these returns in a matrix. This is repeated for the number of simulations **m**.

Script 6.2 –Code for calculating returns and storing them

```
RS = (S(j,end)/S(j,1)-1); %Calculates the return of the stock index
RV = (V(j,end)/V(j,1)-1); %Calculates the return of V
RU = (U(j,end)/U(j,1)-1); %Calculates the return of U

G(1,j)= RV; %Matrix that stores the return of V
G(2,j)= RU; %Matrix that stores the return of U
G(3,j)= RS; %Matrix that stores the return of the stock index
G(4,j)=RS*2; %Matrix stores twice the return of S
```

Script 6.2 shows the part of the script that calculates and stores the return for a single simulation. Description of each line is given by the right side green text. After the simulations have been run **m** times the program computes the expected return and standard deviation of **S**, **U** and **V** for the time horizon using the stored return data.

Script 6.3-Code for computing M-squared measures.

```
meanRV = mean(G(1,:)); %Calculates Expected return of V
meanRU = mean(G(2,:)); %Calculates Expected return of U
meanRS = mean(G(3,:)); %Calculates Expected return of index

stdRV = std(G(1,:)); %Calculates the standard deviation of V
stdRU = std(G(2,:)); %Calculates the standard deviation of U
stdRS = std(G(3,:)); %Calculates the standard deviation of index

SharpeRV=(meanRV-(exp(r*T)-1))/stdRV; %Calculates the Sharpe ratio of V
SharpeRU=(meanRU-(exp(r*T)-1))/stdRU; %Calculates the Sharpe ratio of U
SharpeS=(meanRS-(exp(r*T)-1))/stdRS; %Calculates the Sharpe ratio of index

M2U=((SharpeRU-SharpeS).*stdRS)*100 %Calculates the M^2 of U
M2V=((SharpeRV-SharpeS).*stdRS)*100 %Calculates the M^2 of V
M2D=M2U-M2V %The difference between M2u and M2v
```

Script 6.3 shows how the calculated returns and standard deviations are further used to compute the Sharpe ratio of **S**, **U** and **V**. The Sharpe ratios are in turn used to compute the M-squared measures of **U** and **V**, **M2U** and **M2V**, with **S** as the benchmark. **MD2** is calculated as the difference between **M2U** and **M2V**.

Script 6.4-Code for calculating estimated expected return and standard deviations.

```
[Vmean, Vstd, Vlowbnd, Vupbnd] = meanstd(G(1,:)); %Estimates V's expected return w/95% CI
[Umean, Ustd, Ulowbnd, Uupbnd] = meanstd(G(2,:)); %Estimates U's expected return w/95% CI
[Smean, Sstd, Slowbnd, Supbnd] = meanstd(G(4,:)); %Estimates S's double expected return
w/95% CI

[stdRV, lowStdRV, upStdRV] = stdconf(G(1,:)); %Estimates V's standard dev. w/95% CI
[stdRU, lowStdRU, upStdRU] = stdconf(G(2,:)); %Estimates U's standard dev. w/95% CI
[stdRS, lowStdRS, upStdRS] = stdconf(G(4,:)); %Estimates S's double standard dev. w/95% CI
```

Script 6.4 shows the final part of the MATLAB script that is used to estimated expected returns and standard deviations, with 95% CIs, for \mathbf{U} , \mathbf{V} and double that of \mathbf{S} . The *meanstd.m* and *stdconf.m* are functions. Both these functions are given in the appendix.

Because this analysis considers four values of both μ and σ and six different time horizons the script is run a total of $4 * 4 * 6 = 96$ times.

The main MATLAB script was authored with the help and guidance of supervisor Valeri Zakamouline. He is the sole author of the two function scripts *meanstd.m* and *stdconf.m*.

Chapter 7: Analysis of results

7.1 Performance analysis of U: without transaction costs

The **M2U** values in table 7.1 are all negative apart from the daily values which are all equal to zero. This implies that the daily Sharpe ratios of **U** are equal to the Sharpe ratios of the benchmark no matter the μ - and σ -values. There is of course a possibility that the **M2U**-values actually vary with the different μ - and σ -values, but that the changes are so small that the numbers are rounded up or down during simulations.

Figure 7.1- The relative performance of U over time.

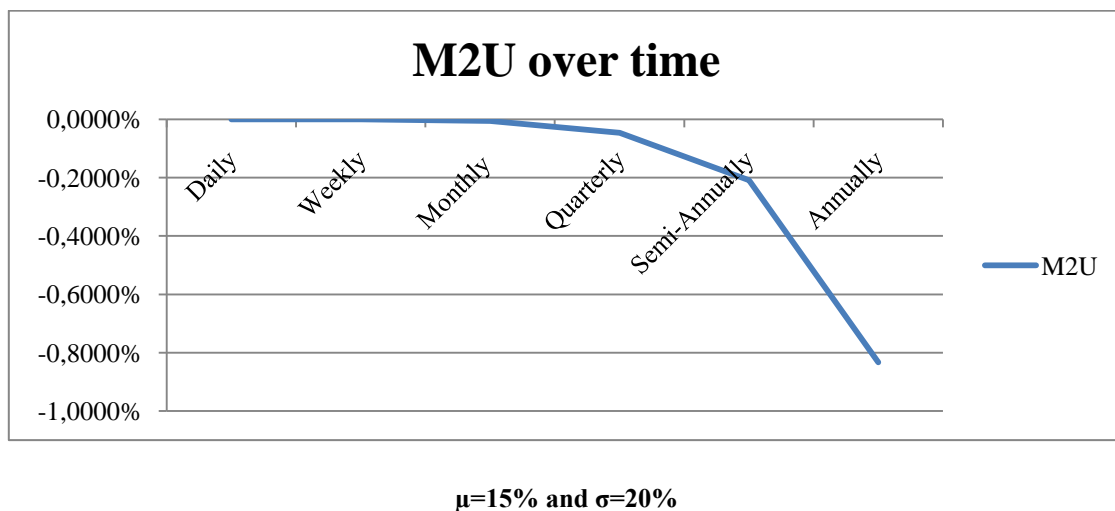
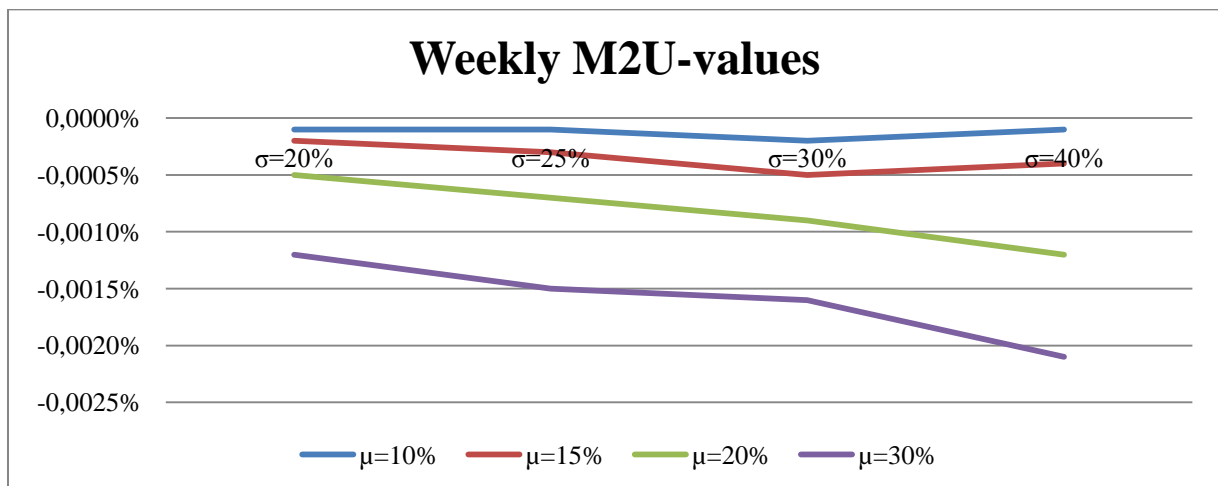


Figure 7.1 shows the general trend of the **M2U** values over time. The shape of the curve in figure 7.1 is the same over time for all μ - and σ -values. However, one cannot compare **M2U**-values between time horizons. It is rational that, for instance, the expected return for weekly-period is less than the monthly return. As such it is intuitive that the difference between adjusted expected return and expected return of the benchmark has a greater nominal value, when the time horizon is longer. However, one can conclude that the performance of **U** is negative for all other time horizons than daily, and that the nominal value between time horizons becomes more negative for longer periods.

Figure 7.2-The weekly relative performance of U.



All values of μ and σ .

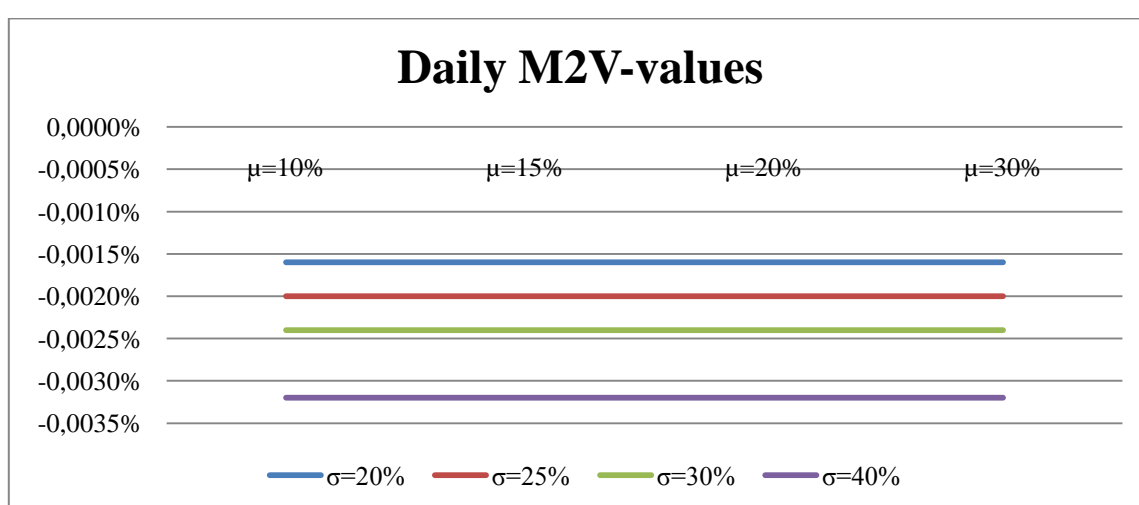
All weekly **M2U**-values in table 7.1 show a clear trend for higher μ -values: The weekly performance declines for higher values of μ , regardless of the value of σ . However, the effect of different σ -values is more ambiguous. Figure 7.2 illustrates **U**'s weekly values for all the four different μ -values and how the σ affects the performance. For the two highest values of μ , 30% and 20%, the performance shows a clear trend of decline with higher σ -values. However for the lowest two μ -values, 10% and 15%, the trend does not follow. For $\mu=10\%$ the **M2U**-values are constant at -0,0001% for all σ -values except at $\sigma=30\%$, where **M2U**=-0,0002%. For $\mu=15\%$, the **M2U**-values show a clear trend for higher σ -values up until between $\sigma=30\%$ and $\sigma=40\%$, where the **M2U**-value increases. It should be noted that the performance at $\sigma=40\%$, is worse than those of $\sigma=20\%$ and $\sigma=30\%$.

U's performance for monthly, quarterly, semi-annual and annual time horizons all show the same two trends with respect to μ and σ . The performance deteriorates for both higher values of μ and σ .

7.2 Performance analysis of V: With transaction costs

The **M2V**-values in table 7.2 are all negative, regardless of time horizon and μ - and σ -values. The daily **M2V**-values are all negative, but appear only to be negatively affected by greater values of σ . Paired with any value of μ , the **M2V**-values becomes increasingly negative with higher σ -values. However, the daily **M2V**-values appear to be unaffected by different μ -values, as there are no changes in **M2V**-values for different μ -values paired with any value of σ .

Figure 7.3-The Daily M2V-values



All μ - and σ -values.

Figure 7.3 shows how all the daily **M2V**-values are only affected by the value of σ . The daily performance declines for higher σ -values, but are all unaffected by the μ . For weekly, monthly, quarterly, semi-annual and annual time horizons the **M2V**-values are negatively affected by both higher values of σ and μ .

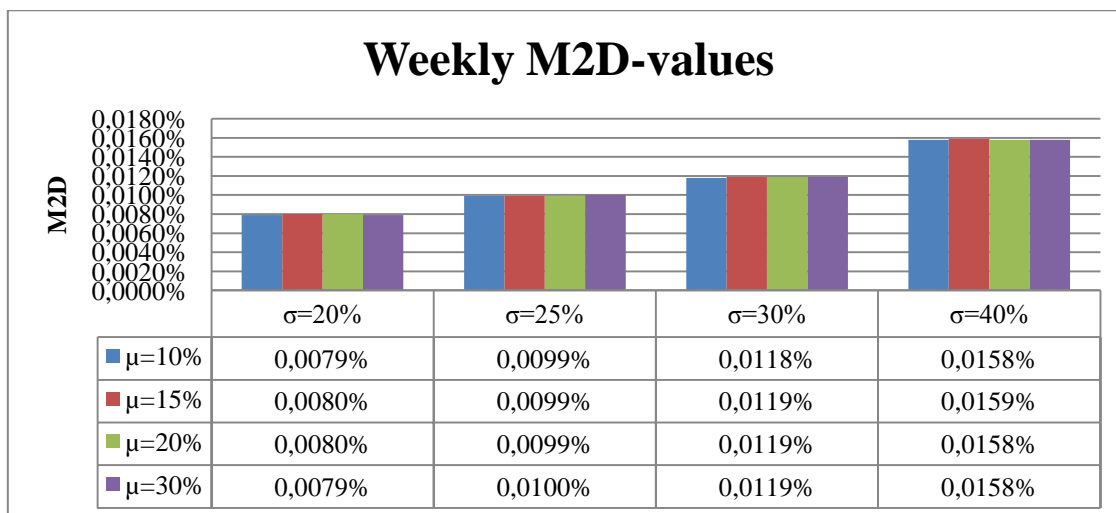
7.3 Comparison of U and V: The effect of transaction costs on performance.

The **M2D**-values in table 7.3 are all positive. This means that the performance of **U** is greater than the performance of **V** for all time horizons and values of μ and σ . The daily differences in performance appear to only be affected by the value of σ . The difference in performance becomes greater for higher σ -values, meaning that the relative performance of **U** becomes increasingly better (or less worse) than that of **V**. This is intuitive from the analysis of daily

performance of **U** and **V**. **U**'s daily performance appears unaffected by both μ and σ , while **V**'s daily performance was only (negatively) affected by higher σ -values.

The weekly performance difference shows a clear trend of increasing with greater values of σ . However the impact of μ -values is more ambiguous.

Figure 7.4-Weekly M2D-values



All values of μ and σ .

It appears that the decreases in performance for both **U** and **V** are nominally equal for higher values of μ , in some instances. This explains why the **M2D**-values appear to be constant for higher μ -values.

Monthly, quarterly, semi-annual and annual **M2D**-values all show similar trends for higher σ -values. The difference in performance becomes bigger for higher σ -values. However, the difference in performance, for the same time horizons, shows a small decrease for higher values of μ . Keeping in mind that both **U** and **V** both have decreasing performance for higher μ -values, the effect of higher μ -values on the difference in performance suggests that the decrease in performance for **V** is relatively smaller than the decrease in performance of **U**.

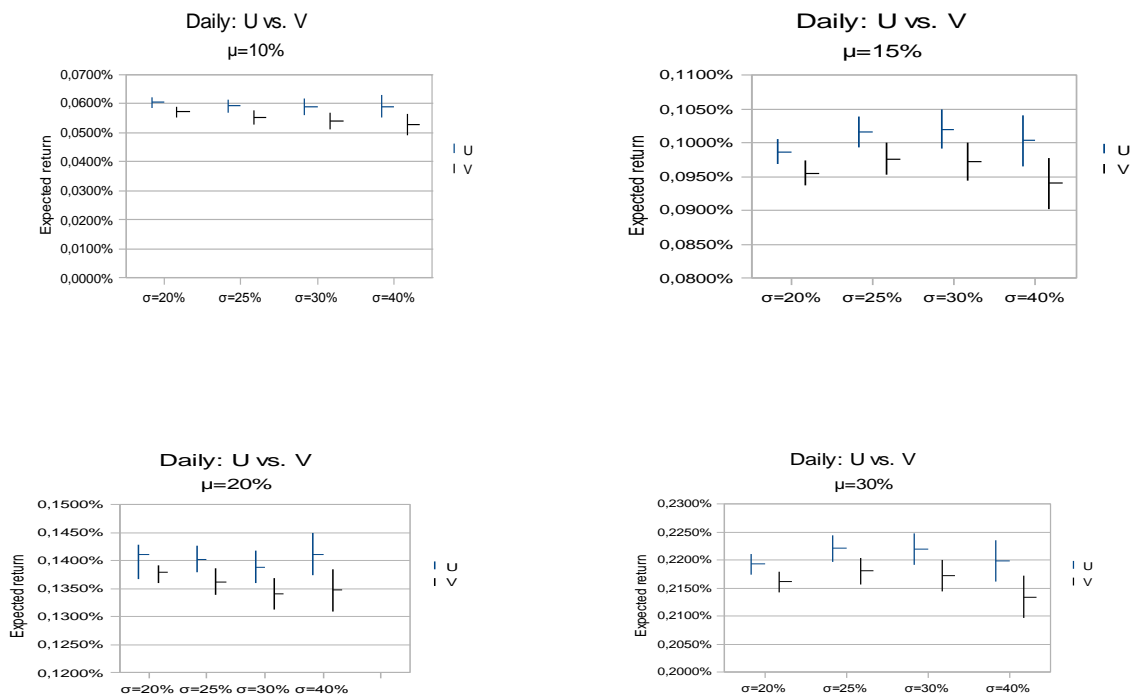
7.4 Analysis of expected returns

In this section the estimated expected returns of **U** and **V** are compared for a daily, weekly and monthly time horizon. The estimated expected returns are also analyzed for the different values of σ and μ , in order to analyze if differences in such values have any impact on the performance of **U** and **V**.

7.4.1 Analysis of daily expected returns

The estimated daily expected return of **U** and **V** are given in table 7.4.

Figure 7.5- The daily expected returns of **U** and **V** vs. σ .



The estimated daily expected returns of both **U** and **V** do not seem to follow any clear trend with regards to different σ -values, as all four diagrams in figure 7.5 shows. The 95% CI of **U**'s estimated expected returns all overlap for any value of σ in all of the four diagrams. This is also the case for all 95% CI of **V**'s estimated expected return. This means that different σ -values do not seem to have a significant effect on the estimated daily expected returns of either **U** or **V**: We cannot say that the daily estimated expected returns of either fund become significantly greater or less for different σ -values.

The effect of μ -values is clearer. The estimated expected returns of both **U** and **V** become significantly higher as the value of μ increases. This trend holds for all values of σ .

When comparing the daily estimated expected returns of **U** with the corresponding estimated expected returns of **V**, we observe that **U** has the greatest estimated expected return for any pair of μ and σ . However, we also see that the 95% CIs of the expected returns of **U** and **V** overlap for any pair of μ and σ . The analysis of the daily estimated expected returns therefore shows that there are no significant difference in estimated expected return between **U** and **V**.

7.4.2 Analysis of weekly expected returns

The weekly estimated expected returns of **U** and **V** are given in tables 7.5. The weekly estimated expected returns of both **U** and **V** do not appear to follow any clear trends with regards to different σ -values. **U**'s estimated expected returns are not significantly different for different values of σ . This holds all μ -values. For **V**'s estimated expected returns, there is only one instance where the estimated expected return is significantly different than the other expected returns:

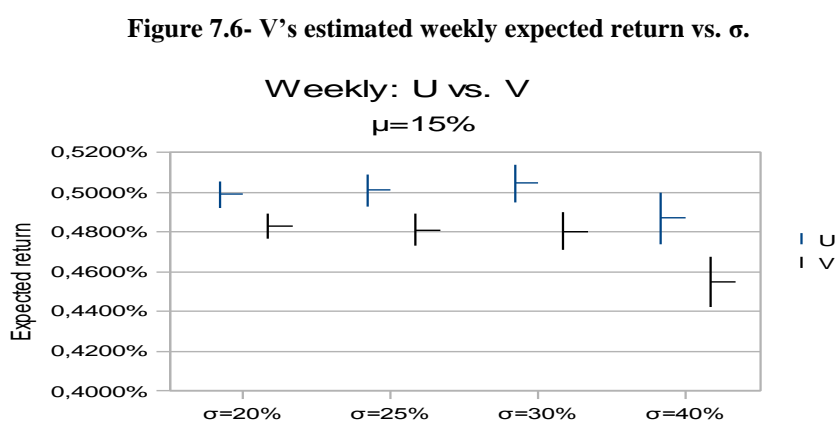


Figure 7.6 shows how the estimated expected return at $\sigma=40\%$ is significantly less than the estimated expected return for all the other three values of σ , when paired with $\mu=15\%$. Other than this exception, there are no other cases of **V**'s estimated expected return being significantly different for different values of σ , paired with any μ -value.

Like the daily estimated expected returns of **U** and **V**, both funds' estimated expected returns become significantly greater for higher μ -values.

When comparing the estimated expected returns of both funds the data shows that the estimated expected return of **U** is greater than the corresponding expected return of **V**, and that none of the 95% CI overlap for any pair of μ and σ . This means that for a weekly time period, all estimated expected returns of **U** are significantly greater than the corresponding estimated expected returns of **V**.

7.4.3 Analysis of Monthly expected returns.

The monthly estimates for **U** and **V**'s expected returns are given in table 7.6. **U**'s estimated expected return show no significant difference for different σ -values paired with any value of μ . The effect of σ -values on **U**'s estimated expected return does not seem to follow any readable trend. The effects of different μ -values, shows that **U**'s estimated expected return becomes significantly greater for increased μ -values, for any σ -value.

V's monthly estimated expected return show some instances of being significantly different for different σ -values. When paired with $\mu=10\%$, the estimated expected return at $\sigma=40\%$ is significantly less than the estimated expected return at both $\sigma=20\%$ and $\sigma=25\%$. When paired with $\mu=30\%$, the estimated expected return at $\sigma=40\%$ is significantly less than $\sigma=20\%$. Aside from these aforementioned exceptions all other estimated expected returns do not differ significantly with difference in σ , paired with any μ -values. The effect of different μ -values show that **V**'s monthly estimated expected return increases significantly for higher μ -values, no matter the value of σ .

The estimated expected return of **U** is greater than the corresponding estimated expected returns of **V** for any pair of σ and μ . None of the 95% CI overlap, meaning that the monthly estimated expected returns of **U** are significantly greater than the monthly estimated expected returns of **V**.

7.5 Analysis of Standard deviations.

In this section we analyze and compare the estimated standard deviations of **U** and **V** for different values of μ and σ , in order to uncover how such values affects the standard deviations of the two funds. The estimated standard deviations of **U** and **V** are compared to determine if there are any significant differences between the two.

7.5.1 Analysis of Daily standard deviations.

The estimates for daily standard deviations of **U** and **V**, with 95% CI, for different values of μ and σ are given in table 7.7.

Figure 7.7-Estimated daily standard deviations of U and V vs. μ -values.

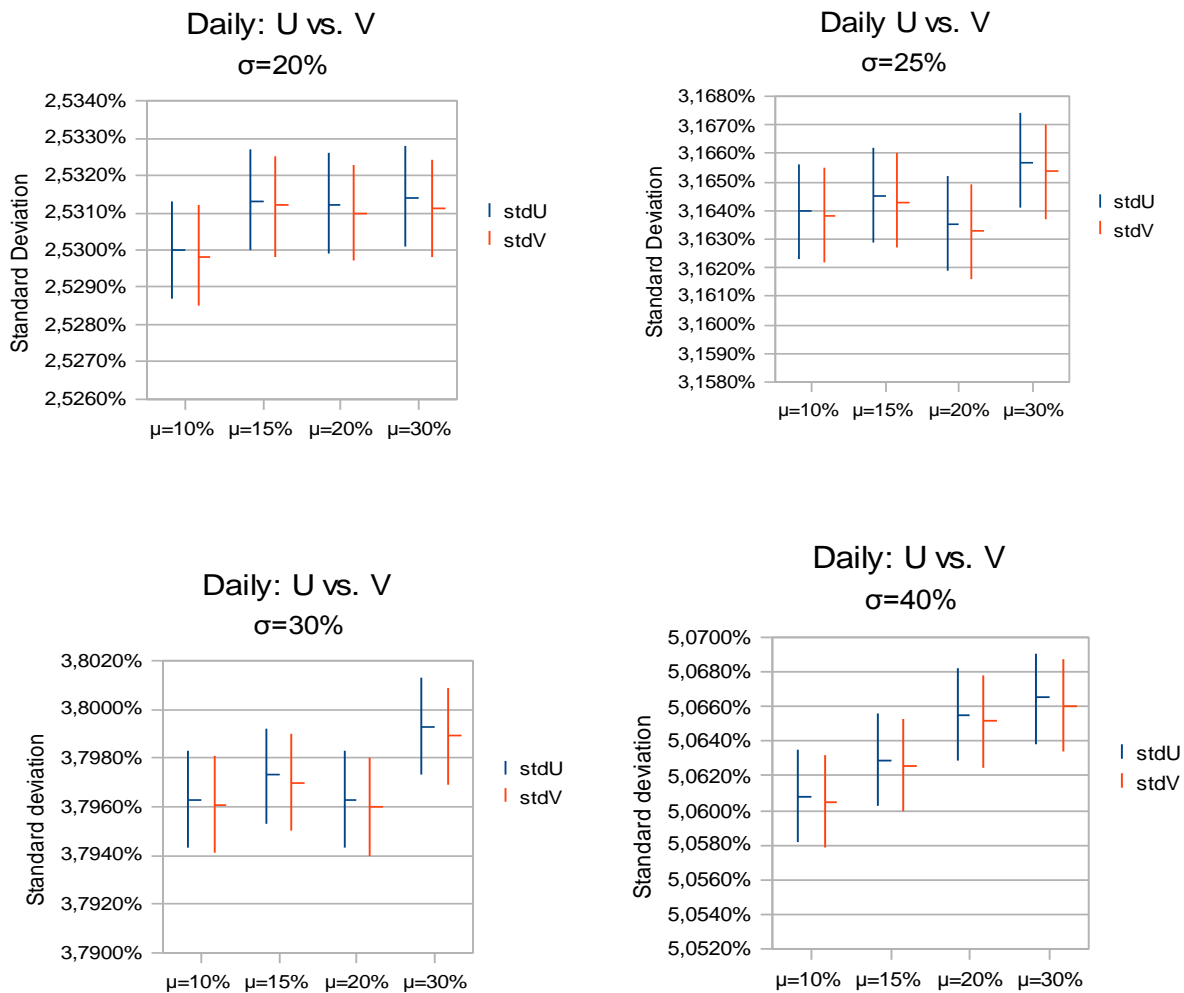
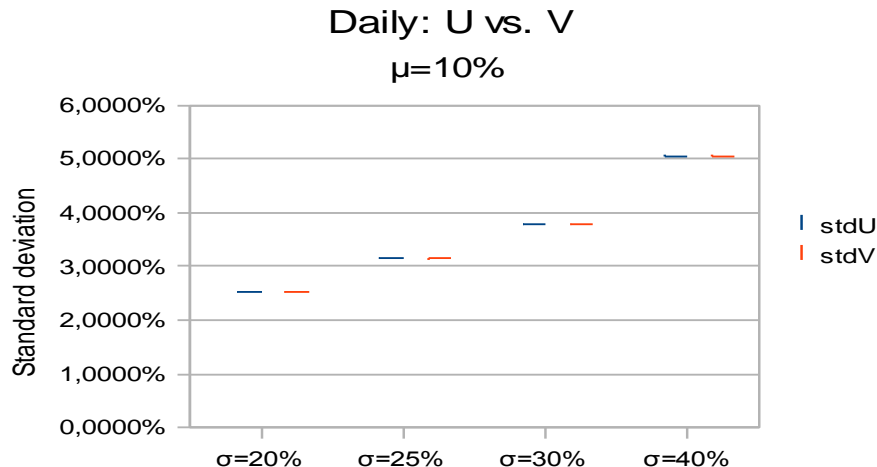


Figure 7.7 shows the estimated daily standard deviations of **U** and **V** against all μ -values paired with all σ -values. The effect of μ -values on **U**'s estimated daily standard deviation show for the most part no clear trend. For $\mu=10\%$, $\mu=15\%$ and $\mu=20\%$ the trends are different for different σ -values. However, for all σ -values, **U**'s estimated daily standard deviation is highest for $\mu=30\%$. There does not appear to be any significant difference in **U**'s estimated standard deviation for different μ -values, as they all overlap when paired with any σ -value. The only exception is for $\sigma=40\%$, where the 95% CIs of **U**'s estimated standard deviations for $\mu=10\%$ and $\mu=30\%$ do not overlap.

The estimated daily standard deviations of **V** do not appear to follow any clear trend with regards to different values of μ . All of the 95% CIs overlap with the exception of the 95% CI of $\mu=10\%$ and $\mu=30\%$, paired with $\sigma=40\%$, which shows that the estimated standard deviation at $\mu=30\%$ is significantly higher than for $\mu=10\%$.

Figure 7.8-The effect of higher σ -values on the estimated standard deviations of **U and **V**.**



The effect of higher σ -values on both **U**'s and **V**'s estimated standard deviations, for $\mu=10\%$ is shown in figure 7.8. Both funds have significantly higher estimated standard deviations for higher values of σ . This holds for any value of μ .

When comparing the estimated daily standard deviations of **U** and **V**, we see that the estimated standard deviation of **U** is always a little higher than the corresponding standard

deviation of V , for any pair of σ and μ . However, we also observe that all of the 95% CIs of U 's and V 's standard deviations overlap, meaning that the difference are not significant.

7.5.2 Analysis of Weekly Standard deviations

The estimates of the weekly standard deviations of U and V are given in table 7.8.

Figure 7.9-Estimates of weekly standard deviations of U and V vs. μ -values.

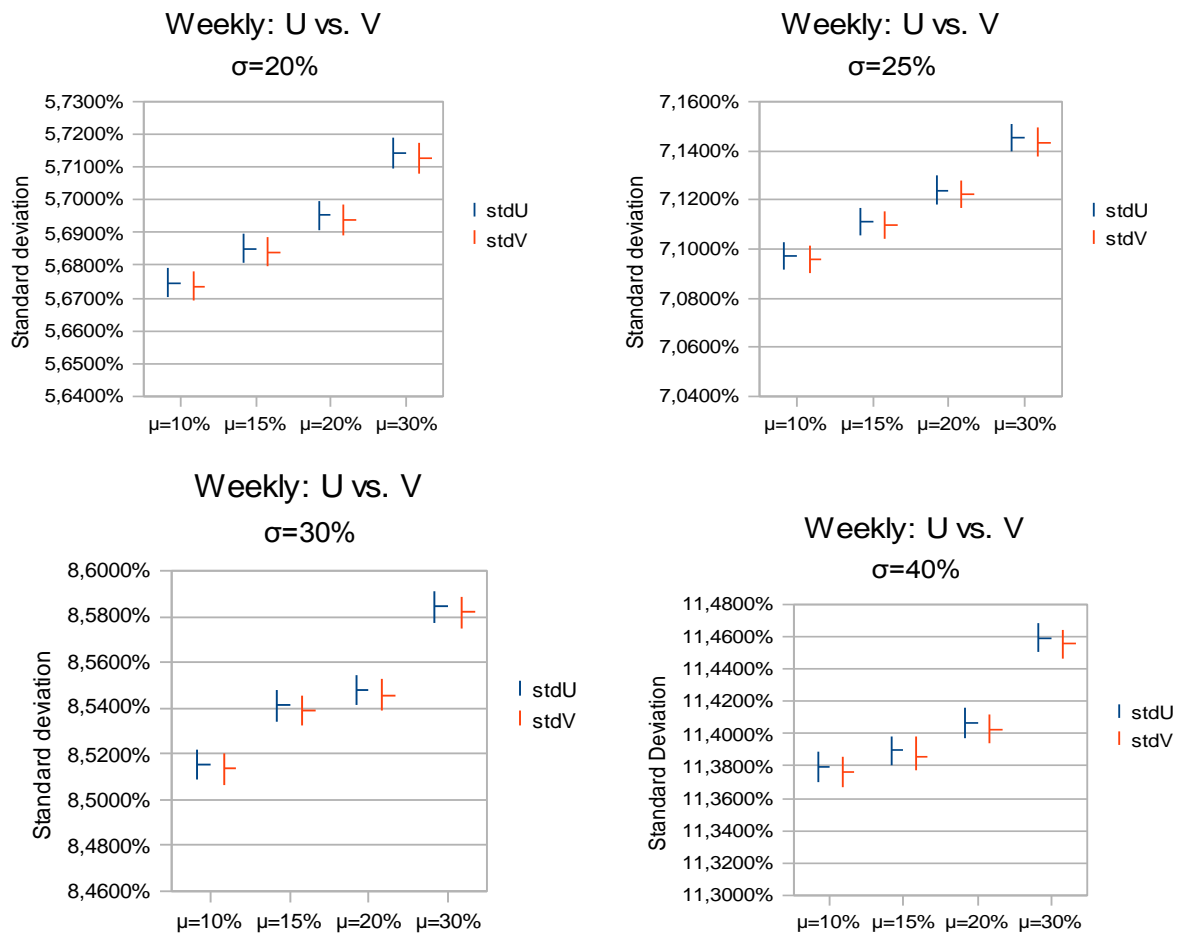


Figure 7.9 shows the estimated weekly standard deviations of U and V against the μ -values, paired with all four values of σ . The estimated weekly standard deviations of both U and V show increases for higher μ -values, paired with any σ -value. For $\sigma=20\%$ and $\sigma=25\%$ these increases are significant as none of the 95% CIs of estimated standard deviations overlap for higher μ -values. This holds for both U and V . For $\sigma=30\%$ there is an overlapping of 95% CIs for both U and V between $\mu=15\%$ and $\mu=20\%$. For $\sigma=40\%$ the 95% CIs of both U and V overlap between $\mu=10\%$ and $\mu=15\%$, and a second overlapping occurs between $\mu=15\%$ and

$\mu=20\%$. In all four cases displayed in figure 7.9 the estimated standard deviations of both **U** and **V** at $\mu=30\%$ are significantly greater than the estimated standard deviations for the three lower μ -values.

The effect of increased σ -values is the same on the weekly estimated standard deviations as it is on the daily estimated standard deviations: The estimated standard deviations of both **U** and **V** grows for higher σ , and none of the 95% CIs intervals between σ -values overlap, meaning that the estimated standard deviations of both **U** and **V** increase significantly for higher σ -values.

The estimated weekly standard deviations of **U** are all a little higher than the corresponding standard deviations of **V**. However, like the daily estimates, the 95% CI of **U**'s and **V**'s estimated standard deviations all overlap, indicating that the difference in estimated weekly standard deviations are not significant.

7.5.3 Analysis of Monthly standard deviations.

The estimated monthly standard deviations of **U** and **V** with 95% CIs are given in tables 7.9.

The estimated monthly standard deviations of **U** all show an increase in value with higher μ -values. This trend holds for all four values of σ . The increase in **U**'s estimated monthly standard deviations for higher μ -values is significant, as none of the 95% CIs overlap. The trend is the same for estimated monthly standard deviations of **V**. All estimated standard deviations increase significantly for higher values of μ , regardless of the value of σ .

The effect, on estimated standard deviations of **U** and **V** for higher σ -values are the same on the daily and weekly values: The estimated standard deviations increase significantly for higher values of σ .

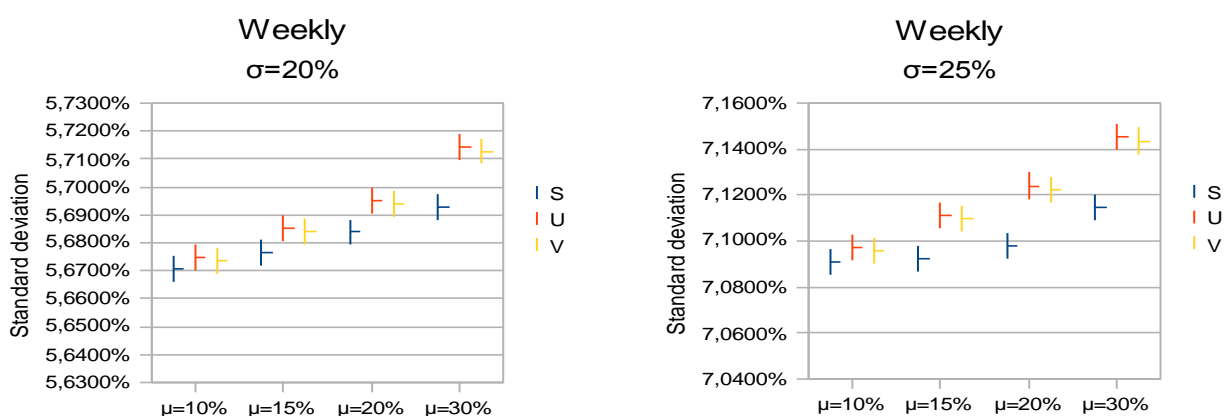
The estimated monthly standard deviations of **U** are all a little greater than the corresponding standard deviations of **V**, however, like the daily and weekly standard deviations, all 95% CIs overlap between corresponding estimated standard deviations of **U** and **V**, meaning that the difference is not significant.

7.5.4 Analysis of estimated standard deviation of the benchmark.

The estimated standard deviation of twice the expected return of the benchmark, **S**, is given in table 7.10, for a daily, weekly and monthly time horizon. Because the M-squared measure also depends on the standard deviation of the funds, it is of interest to research how the estimated standard deviations of the benchmark are compared with the corresponding estimated standard deviations of **U** and **V**. The daily double estimated standard deviations of **S**, in table 7.10, are compared with the corresponding values of **U** and **V** in table 7.4. On a daily basis the estimated standard deviations of **S** are all a little small than the values of **U** and **V**. However, all 95% CIs, between **S**, **U** and **V**, overlap indicating that the differences are not significant.

The estimated weekly double standard deviations of **S** are compared with the corresponding values for **U** and **V** in table 7.5. The weekly values of **S** compared with the corresponding values of **U** and **V** shows that, the estimated double weekly standard deviations of **S** are all smaller than the values of **U** and **V**, for any pair of σ and μ . These differences are for the most part significant. The differences that are not significant happen for low values of both σ and μ .

Figure 7.10-The weekly estimated standard deviations of S (double), U and V for $\mu=10\%$ and $\mu=15\%$.



The diagram to the left in figure 7.10 shows how the estimated double standard deviations of **S** is not significantly different from the corresponding values of **U** and **V**, for $\mu=10\%$ and

$\mu=15\%$, when paired with $\sigma=20\%$. The diagram to the right shows the estimated double standard deviation of **S** is not significantly less than the corresponding values for **U** and **V**, for $\mu=10\%$ paired with $\sigma=25\%$,

The monthly estimated standard deviations of **U** and **V** are given in table 7.6. Comparing these estimated standard deviations with the benchmarks shows that the benchmark has a significantly smaller double estimated standard deviation, than the two funds for all values of μ and σ , as its 95% CIs do not overlap with the corresponding 95% CIs of the two funds.

The data in table 7.10 shows that for all three time horizons the estimated double standard deviation becomes greater for higher σ -values. It also shows that the estimated double standard deviation generally becomes greater for higher μ -values, for all three time horizons. There are two exceptions; one for a daily and one for a weekly time horizon. The daily happens between $\mu=15\%$ and $\mu=20\%$ for $\sigma=25\%$ and the weekly happens between $\mu=15\%$ and $\mu=20\%$ for $\sigma=30\%$. In both cases the estimated double standard deviation decreases a bit, but these decreases are very small.

7.6 Accounting for the Management fee.

At the inception of both Bull funds, both Handelsbanken and DnbNOR set the management fee to 0,8% per annum of the total value of the funds. While the management fee is represented per annum, it is deducted on a daily basis from the funds' assets. It is therefore of interest to research if the daily deductions affects the performance of the fund significantly. Table 7.11 contains the daily expected returns with 95% CI of a fund that both accounts for the bid-ask spread and the management fee of 0,8 per annum. The fund is called **V***. The simulations are run for the same four values of both μ and σ . When we analyzed the impact of transaction costs on the daily estimated expected returns we found that the estimated expected returns of **U** and **V** did not differ significantly. However when we compare the daily estimated expected returns of **U**, in table 7.4, with the daily estimated expected returns **V***, we see that the estimated expected returns are significantly different. All daily estimated expected returns of **V*** are less than the corresponding estimated expected returns of **U**, and none of the 95%

CI overlap. The combined effect of both transaction costs and the management fee seems therefore, on a daily basis, to reduce the estimated expected return significantly.

7.7 The effect of the risk-free rate on Expected returns

In the presentation of our model in chapter 6 we showed how the expected return of the Bull fund is negatively affected by a positive risk-free rate. Because the basis of the analysis in this paper is to compare the fund with and without transaction costs, it is of interest to determine the possible effect the risk-free rate might have on our results.

Figure 7.11- Weekly expected return: U vs. V vs. S (double) for positive risk-free rate.

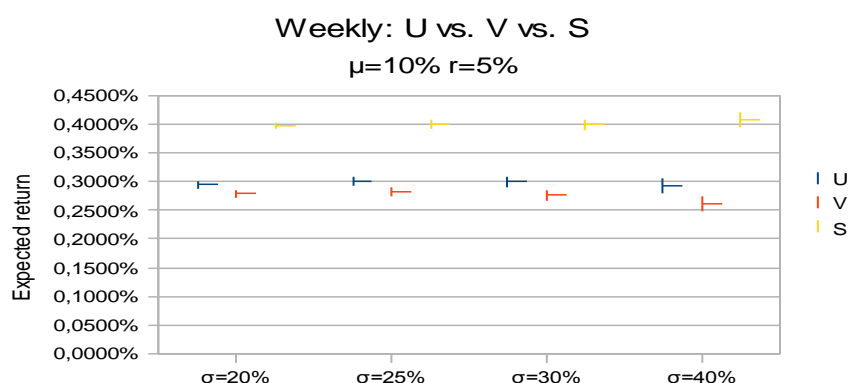


Figure 7.11 compares the weekly estimated expected returns of the two funds to twice the expected return of the benchmark for $\mu=10\%$, $r=5\%$ and all four values of σ . The data for the benchmark S, is given in table 7.12. As previously uncovered, the weekly estimated expected returns of V is significantly less than those of U, but both funds' estimated expected return is also significantly less than twice the expected return of the benchmark, as shown in figure 7.8.

Figure 7.12- Weekly expected return: U vs. V vs. S (double) for zero risk-free rate.

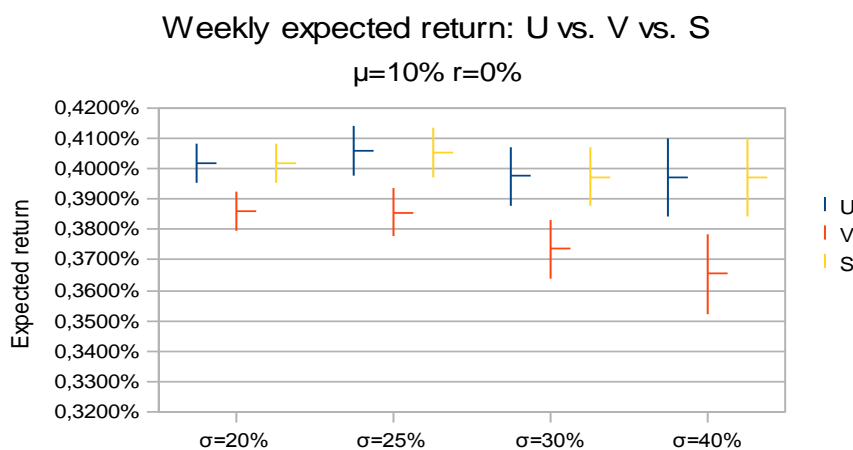


Figure 7.12 compares the weekly estimated expected returns of **U** and **V** with twice the estimated expected return of **S**, for $\mu=10\%$ and $r=0\%$. The data of all three funds are given in table 7.13. As the figure clearly illustrates, the estimated expected returns of **U** and twice that of **S**, are now almost equal for all four σ -values. All of the 95% CI overlap, indicating that there is no significant difference between the estimated expected return of **U** and twice that of **S**. At the same time, the estimated expected returns of **V** are significantly less than those of **U** (and **S**), with no overlapping 95% CI. The level of the risk-free rate does not influence the conclusions of significant difference between the estimated expected returns of **U** and **V**.

7.8 Analysis summary: Discussion of results

First, it is important to address the shortcomings of the M-squared measures. The measures are unlike; the estimated expected returns and standard deviations, not presented in confidence intervals. The expected returns and standard deviations that go into the computations of the M-squared measures are only point estimates. Therefore when trying to explain the negative performance of both **U** and **V**, we can only consider the point estimates of expected returns and standard deviation, and not whether these are significantly different from one another or the corresponding values of the benchmark. It also means that it is impossible to tell whether decreases in performance for higher μ - and σ -values are actually significant.

Remember from chapter 4.3.4 that it was shown that the M-squared measure of a Bull fund would be equal to zero if the fund delivered twice the expected return of the benchmark at twice the standard deviation of the benchmark. In our analysis we have seen that, apart from **U**'s daily performance, both **U** and **V** have a negative performance relative to the underlying benchmark, for all other time horizons. The effect of a positive risk-free rate in chapter 7.7 explains part of this underperformance, as a positive risk-free rate decreases the expected returns of **U** and **V**, while the expected return of the benchmark remains unchanged. The second contributor to underperformance on the Bull funds' part appears to be the level of the estimated standard deviations. When the estimated standard deviations of the funds were compared to that of twice the estimated standard deviations of the benchmark, in chapter 7.5.4, both funds had greater estimated standard deviations compared with the benchmark. This was the case for all three periods considered.

The analysis of estimated expected returns and standard deviations offers no clear cause to why the weekly and monthly performance of **U** and **V**, generally deteriorates for higher μ - and σ -values. Consider the σ -values first. None of the two funds' estimated expected returns show any clear trends with regards to different σ -values on a weekly and monthly basis. However, both funds have significantly increases in their estimated standard deviation for higher σ -values. One possible explanation might be that the funds' estimated standard deviations grows bigger for higher σ -values compared with the estimated standard deviations of the benchmark. As previously stated, double the estimated standard deviations of the benchmark are significantly lower than those of the funds' for a weekly and monthly period. Another possible explanation can be related to the actual way of which the M-squared measure is calculated. Consider the M-squared measure of **U** to be given by (4.23):

$$M_U^2 = (SR_U - SR_S) * \sigma_S$$

If the differences in Sharpe ratios between **U** and **S** are actually constant for regardless of σ -value, then the M-squared measure of **U** will only be driven by the size of the σ . To relate this to our analysis, it can mean that the negative difference between Sharpe ratios of **U** and **S** are in fact constant for higher σ -values, then nominal increase in negative performance is in fact only driven by $\sigma(\sigma_S)$. Therefore, without the possibility of confidence intervals it is hard to tell if the actual performance deteriorates for higher σ -values.

However, the calculation procedure of the M-squared measure does not imply that increases/decreases can occur for constant differences in Sharpe ratios, for different μ -values. The estimated standard deviations of **U** and **V** all show a significant increase for increases in the μ -value, as do the estimated double standard deviation of the benchmark. It is therefore possible that greater increase in estimated standard deviations on the funds' part is to blame for the decrease in performance, i.e. M-squared measure, for higher μ -values.

In section 3.2.5 it is shown how the bid-ask spread theoretically affects both the expected return and standard deviation of an investment. Based on the analysis and comparison of the

estimated expected returns and standard deviations of the two funds, this theory seems to hold. The estimated expected returns of **V** are less than the corresponding estimated expected returns of **U**. As the analysis shows, these differences are not significant on a daily basis, but on a weekly and monthly basis, the estimated expected returns of **V** are significantly less than those of **U**. The analysis and comparison of estimated standard deviations are also in line with the theory. The estimated standard deviations of **V** are less than those of **U**. However none of these differences are significant. The result from comparison of these estimated values helps to explain the positive **M2D**-values, i.e. why **U** outperforms **V** in all instances. The bid-ask spread causes the relative difference in expected returns to be greater than the relative difference in standard deviations, resulting in **V**'s performance being worse than that of **U**.

However, explaining why the difference in performance, **M2D**, becomes greater for higher σ -values is hard based on the analyzed data at hand. The calculation of the **M2D**-values might also suffer under the same flaws as the other M-squared measures, when different σ -values are considered. By taking the difference between the M-squared measures of **U** and **V**, the **M2D**-values are calculated:

$$M2D = (SR_U - SR_S) * \sigma_S - (SR_V - SR_S) * \sigma_S = (SR_U - SR_V) * \sigma_S$$

Again, based on the calculation it is in fact possible that the differences between Sharpe ratios are constant for σ -values, so that it is only the size of the σ that determines the **M2D**-value.

The cause of the small decrease in monthly **M2D**-values for increased μ -values is also hard to establish. The monthly estimated expected return and standard deviation of **U** and **V** show significant increases for higher μ -values. There are many possible explanations for this. One possibility is that the estimated standard deviation of **V** grows less than the estimated standard deviation of **U**. Another possibility is that the estimated expected return of **V** grows more than the estimated expected return of **U**. A third possibility is that the estimated return of **V** grows bigger, while the estimated standard deviation grows slower. The possibilities are many.

However, without confidence intervals it is hard to say whether this decrease in **M2D**-values for higher μ -values is in fact significant.

Chapter 8: Conclusion

This master thesis set out to investigate how the transaction costs associated with the daily rebalancing might affect the performance of Bull fund, and whether the magnitude of such effects might depend on the size of expected returns and standard deviations of the Bull fund's benchmark. In order to research such questions, simulations of price paths of a benchmark stock index and two Bull funds, one with and one without transaction costs, have been carried out for different time horizons and values of the benchmark's annual expected return and annual standard deviation. The results from such simulations have then been used to compute and estimated M-squared measures for the two Bull funds using the underlying stock index as a benchmark. In order to probe the M-squared results, simulation runs have also been used to estimate expected returns and standard deviations with confidence intervals for some of the selected time horizons, i.e. daily, weekly and monthly. The results for the Bull fund with and without transaction costs have been compared in order to uncover differences and to check if such difference are statistically significant. Analysis have also been done to compare the estimated standard deviations of the two funds relative to twice that of the benchmark, as well as the impact of the risk-free rate and the management fee on the expected returns of the funds.

The results from analyzing the M-squared measures show that the Bull fund underperforms the benchmark for all time horizons except daily, even without transaction costs. The results from the M-squared analysis also show that the performance generally becomes increasingly negative for higher values of the benchmarks expected return and standard deviation, both with and without transaction costs. The negative performance with transaction costs are always more negative than without transaction costs.

Problem 1:

The results of the analysis seem offer some explanation of the first research question:

- *If any, what are the effects of transaction costs on the performance of a Bull fund?*

The comparison of the M-squared measures shows that the Bull fund with transaction costs has a poorer relative performance to the benchmark, compared with the fund without transaction costs, for all time horizons and values of annual expected return and standard deviation of the benchmark. This means that the transaction costs affects the Sharpe ratio of the fund negatively, causing the Bull fund to have lower reward-to-variability. The analysis of the daily, weekly and monthly estimated expected returns and standard deviations shows two things: One, the estimated expected returns of the Bull fund with transaction costs are all lower than the corresponding estimated expected returns of the Bull fund without transaction costs. Two, the estimated standard deviations of the Bull Fund with transaction costs are always lower than the corresponding estimated standard deviations of the Bull fund without transaction costs. It seems therefore that transaction costs reduce both the expected return and standard deviation of the Bull fund, and that the relative reduction in expected returns are greater than the relative reduction of standard deviation. This causes M-squared measure to worsen relative to that of the fund without transaction costs. Note however, that the differences in estimated standard deviations are not statistically significant for any of the three aforementioned time horizons. The differences in estimated expected returns show that the estimated expected returns for weekly and monthly time horizons are statistically significantly different, while the daily differences are not. We therefore conclude that the transaction costs over daily, weekly and monthly time periods do not significantly affect the standard deviation. However, transaction costs do significantly reduce the expected returns over weekly and monthly time horizons, but not for a daily time horizon.

The results would indicate that investors that hold a Bull fund for longer than a day will have the expected return of his investment affected by the transaction costs. However, investors must also be aware of the management fee that is deducted from the fund's values on a daily basis. The results in Chapter 7.6 shows how the estimated daily expected return is significantly lower compared with a fund with no costs, when both the bid-ask spread and management fee is considered. Positive risk-free rates will affect the expected return negatively, but as the results in Chapter 7.7 shows, the weekly differences in expected returns will still be significant, even for a zero risk-free rate.

Problem 2:

The second research problem:

- *If any, does the magnitude of such effects depend on the characteristics, i.e. the size of the expected return and standard deviation, of the benchmark?*

The differences in M-squared measures show that the differences in relative performance increases for higher values of the benchmark's standard deviation. This would indicate that transaction costs worsen the performance for higher values of the benchmarks standard deviation. However, comparing the estimated daily, weekly and monthly expected returns and standard deviations of the two funds against higher values of the benchmark's standard deviation offers no clear explanation to what might cause these differences. Higher values of the benchmark's expected returns seem to affect the differences in M-squared measures for monthly, quarterly, semi-annual and annual time horizons. The difference in performance between the two funds show small declines for increased values of the benchmark's expected return. This would indicate that the effect of transaction costs are reduced for higher values of the benchmark's expected return. Comparing the monthly estimated expected returns and standard deviation of the two funds for different values of the benchmark's expected return, offer no clear explanation as to why differences in relative performance is reduced for higher values of the benchmark's expected return. The analysis regarding the second research problem is therefore inconclusive. Possible explanations are discussed in Chapter 7.8.

While no clear cause is found for the increase in relative performance difference between the two funds, comparing the estimated standard deviations of the two funds to twice the estimated standard deviation of the benchmark offers some explanation as to why both funds underperforms the benchmark. The results from Chapter 7.5.4 shows that the estimated standard deviations of both funds are significantly higher than twice the estimated standard deviation of the benchmark for weekly and monthly time horizons. The daily estimated standard deviations for the two funds are also higher than double the estimated standard deviation of the benchmark, but these differences are not significant.

8.1 Weaknesses of results and further research.

The results acquired in this master thesis are based on simulation runs, which in turn is based on simplifications. The transaction costs associated with the rebalancing of a Bull fund also encompasses brokerage fees, clearing fees and depot costs. The size of the spread might also be subject to change depending on the factors presented in Chapters 3.2.2-3.2.4. The spread size used in our simulations is an average based on the data available at Oslo stock exchange's home page. However, the order books on stock index futures at Oslo stock exchange are closed, so the data for intraday trades are not available to the public.

The method applied to investigating the research problems might not be the optimal, as the analysis is inconclusive with regards to research problem 2. In the discussion in Chapter 7.8 we show how it might be possible that the M-squared measure is only driven by the increase in the standard deviation of the benchmark. Another aspect is that comparing the M-squared measures of the Bull fund with and without costs, means comparing four different variables: The expected returns of both funds, and the standard deviations of both funds. If say, varying the standard deviation of the benchmark has an effect on all these variables, it can be hard to determine the effect transaction costs might have on the fund.

In retrospect, a possible alternate method could be a direct comparison of Sharpe ratios of the two funds as well as to the benchmark, with supplementary comparisons of the estimated expected returns and standard deviations of the funds relative to the estimated expected returns and standard deviations of the benchmark.

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Appendix

Deriving the expected return of a Bull fund without costs:

The expected value of the Bull fund (6.7) at day 1 can be expressed as:

$$E[U_1] = U_0 * (e^{r*\Delta t} + 2 * e^{(\mu-r)*\Delta t} - 2)$$

Where μ is the expected return of the benchmark. The expression in the parenthesis is the growth factor. It can be approximated as:

$$e^{r*\Delta t} + 2 * e^{(\mu-r)*\Delta t} - 2 \approx 1 + r * \Delta t + 2(1 + (\mu - r) * \Delta t) - 2 = 1 + (2 * \mu - r) * \Delta t \quad (1)$$

The expected return of the Bull fund at day n can be written:

$$E[U_n] = U_0 (e^{r*\Delta t} + 2 * e^{(\mu-r)*\Delta t} - 2)^n \quad (2)$$

By combining (1) with (2), the Expected return at day n can be expressed as:

$$E[U_n] \approx U_0 * (1 + (2\mu - r) * \Delta t)^n$$

(Haga&Lindset, 2009)

MATLAB SCRIPT FOR RUNNING SIMULATIONS AND ESTIMATING PERFORMANCE MEASURES, EXPECTED RETURNS AND STANDARD DEVIATIONS.

```

clc,clear,clear all,close all

r=0.0;           %The cc risk-free rate
f=0.00;          %The cc management fee of a Bull fund
spread=0.00316;  %Futures price spread
T=1;             %Length of period measured in years
n=T*250;         %Number of steps in a single simulated path
m=70000;        %Number of simulations
mu=0.10;        %CC expected return of the stock index
sigma=0.30;     %Standard deviation of the stock index
dt=T./n;        % Size of each timeperiod
M=2;            % Bull fund multiple
t=dt:dt:T;      %Time horizon

S0=100;         %Index price at time 0
V0=100;         %Value of Bull fund at time 0
S=zeros(m,n);  %Matrix reserved for Index prices
V=zeros(m,n);  %Matrix reserved for Bull fund values
F=zeros(m,n);  %Matrix reserved for Futures' prices
N=zeros(m,n);  %Matrix reserved for number of futures contracts

S(1:m,1)=S0;    %Assigns the value S0 as the index price at time 0
V(1:m,1)=V0;    %Assigns the value V0 as V's value at time 0
F(1:m,1)=S0.*exp(r.*T); %Assigns the futures price at time 0
U(1:m,1)=V0;    %Assigns V0 as U's value at time 0
N(1:m,1)=(M.*V(1:m,1))./F(1:m,1); %Assigns the number of contracts at time 0
G=zeros(7,m);   %Matrix that stores returns and end value of fund

for j=1:m
    for i=1:n
        R=(mu-0.5.*sigma.^2).*dt+sigma.*sqrt(dt).*randn; %Stock index return per period dt
        S(j,i+1)=S(j,i).*exp(R); %Price path of stock index
        F(j,i+1)=S(j,i+1).*exp(r.*(T-i.*dt)); %Futures price path
        Vt = V(j,i)*( exp(r*dt)+M*(exp(R-r*dt)-1) )*(1-(exp(f*dt)-1)); %Value of V
                                                %less management fee

        N(j,i+1) = M*Vt/F(j,i+1); %Number of contracts per period
        dN = abs( N(j,i+1)-N(j,i) ); %Change in number of contracts
        U(j,i+1) = U(j,i)*(exp(r*dt)+M*(exp(R-r*dt)-1)); %Value of U per period
        V(j,i+1) = Vt-dN*spread*F(j,i+1)/2; %Value of V less spread and
                                                %management fee per period

    end

    RS = (S(j,end)/S(j,1)-1); %Calculates the return of the stock index
    RV = (V(j,end)/V(j,1)-1); %Calculates the return of V
    RU = (U(j,end)/U(j,1)-1); %Calculates the return of U

    G(1,j)= RV; %Matrix that stores the return of V
    G(2,j)= RU; %Matrix that stores the return of U
    G(3,j)= RS; %Matrix that stores the return of the stock index
    G(4,j)=RS*2; %Matrix stores twice the return of S

end

% computations of expected returns and std. deviations of returns
meanRV = mean(G(1,:)); %Calculates Expected return of V
meanRU = mean(G(2,:)); %Calculates Expected return of U
meanRS = mean(G(3,:)); %Calculates Expected return of index

stdRV = std(G(1,:)); %Calculates the standard deviation of V
stdRU = std(G(2,:)); %Calculates the standard deviation of U
stdRS = std(G(3,:)); %Calculates the standard deviation of index

SharpeRV=(meanRV-(exp(r*T)-1))/stdRV; %Calculates the Sharpe ratio of V
SharpeRU=(meanRU-(exp(r*T)-1))/stdRU; %Calculates the Sharpe ratio of U
SharpeS=(meanRS-(exp(r*T)-1))/stdRS; %Calculates the Sharpe ratio of index

M2U=((SharpeRU-SharpeS).*stdRS)*100 %Calculates the M^2 of U

```

```

M2V=((SharpeRV-SharpeS).*stdRS)*100      %Calculates the M^2 of V
M2D=M2U-M2V                             %The difference between M2u and M2v

[Vmean, Vstd, Vlowbnd, Vupbnd] = meanstd(G(1,:)); %Estimates V's expected return w/95% CI
[Umean, Ustd, Ulowbnd, Uupbnd] = meanstd(G(2,:)); %Estimates U's expected return w/95% CI
[Smean, Sstd, Slowbnd, Supbnd] = meanstd(G(4,:)); %Estimates S's double expected return
w/95% CI

[stdRV, lowStdRV, upStdRV] = stdconf(G(1,:)); %Estimates V's standard dev. w/95% CI
[stdRu, lowStdRu, upStdRu] = stdconf(G(2,:)); %Estimates U's standard dev. w/95% CI
[stdRS, lowStdRS, upStdRS] = stdconf(G(4,:)); %Estimates S's double standard dev. w/95% CI

```

MATLAB FUNCTION: *meanstd.m*

```

function [amean, astd, lowbnd, upbnd] = meanstd(x, dt)

% Function [amean, astd, lowbnd, upbnd] = meanstd(x, dt) or
% [amean, astd, lowbnd, upbnd] = meanstd(x)
% The function calculates the (annualized) mean of x, standard
% deviation of x, and 95% confidence interval for mean value of x
% Input: x - vector , dt - time interval
% Output:
% (1) amean - (annualized) mean,
% (2) astd - (annualized) standard deviation
% (3) lowbnd - lower bound of the 95% confidence interval for amean
% (3) upbnd - upper bound of the 95% confidence interval for amean
% If you call [amean, astd, lowbnd, upbnd] = meanstd(x) then it is assumed
% that dt = 1
% If you do not need lower and upper bounds, just call
% [amean, astd] = meanstd(x, dt)

if nargin < 2
    dt = 1; % if we supply only one argument
end

amean = mean(x)/dt; % mean / annual mean

s = std(x); % standard deviation

astd = s/sqrt(dt); % std / annual std

n = length(x); % get the length of vector x
err = s/sqrt(n)/dt; % calculate std. error

lowbnd = amean - 1.96*err;
upbnd = amean + 1.96*err;

```

MATLAB FUNCTION: *stdconf.m*

```

function [astd, lowbnd, upbnd] = stdconf(x)

astd = std(x); %Calculates standard deviation of x
n = length(x); %Number of observations
v = var(x); %Calculates variance of x
lowbnd = sqrt((n-1)*v/chi2inv(0.975, n-1)); %Calculates lower bound of 95% CI
upbnd = sqrt((n-1)*v/chi2inv(0.025, n-1)); %Calculates Upper bound of 95% CI

```

Table 7.1-The M-squared Measure of U: M2U

M2U	Daily	Weekly	Monthly	Quarterly	Semi-Annually	Annually
T	0,004	0,020	0,080	0,240	0,500	1,000
$\mu=10\%$						
$\sigma=20\%$	0,0000%	-0,0001%	-0,0019%	-0,0162%	-0,0708%	-0,2843 %
$\sigma=25\%$	0,0000%	-0,0001%	-0,0020%	-0,0200%	-0,0974%	-0,3577 %
$\sigma=30\%$	0,0000%	-0,0002%	-0,0035%	-0,0247%	-0,1159%	-0,4347 %
$\sigma=40\%$	0,0000%	-0,0001%	-0,0043%	-0,0439%	-0,2172%	-0,7270 %
$\mu=15\%$						
$\sigma=20\%$	0,0000%	-0,0002%	-0,0052%	-0,0454%	-0,2086%	-0,8333%
$\sigma=25\%$	0,0000%	-0,0003%	-0,0062%	-0,0538%	-0,2437%	-1,0566%
$\sigma=30\%$	0,0000%	-0,0005%	-0,0068%	-0,0666%	-0,3064%	-1,2336%
$\sigma=40\%$	0,0000%	-0,0004%	-0,0094%	-0,1075%	-0,4035%	-1,7769%
$\mu=20\%$						
$\sigma=20\%$	0,0000%	-0,0005%	-0,0096%	-0,0915%	-0,3934%	-1,6525%
$\sigma=25\%$	0,0000%	-0,0007%	-0,0115%	-0,1029%	-0,4823%	-1,9306%
$\sigma=30\%$	0,0000%	-0,0009%	-0,0126%	-0,1207%	-0,5522%	-2,2273%
$\sigma=40\%$	0,0000%	-0,0012%	-0,0180%	-0,1756%	-0,7010%	-2,9728%
$\mu=30\%$						
$\sigma=20\%$	0,0000%	-0,0012%	-0,0230%	-0,2207%	-1,0037%	-4,0752%
$\sigma=25\%$	0,0000%	-0,0015%	-0,0265%	-0,2453%	-1,1029%	-4,4860%
$\sigma=30\%$	0,0000%	-0,0016%	-0,0289%	-0,2769%	-1,2356%	-5,1369%
$\sigma=40\%$	0,0000%	-0,0021%	-0,0374%	-0,3520%	-1,5635%	-6,3708%

$r=0.05$, $f = 0$, spread=0.00316

Table 7.2-M-squared measure of V: M2V

M2V	Daily	Weekly	Monthly	Quarterly	Semi-Annually	Annually
T	0,004	0,020	0,080	0,240	0,500	1,000
$\mu=10\%$						
$\sigma=20\%$	-0,0016%	-0,0080%	-0,0337%	-0,1113%	-0,2676%	-0,6730 %
$\sigma=25\%$	-0,0020%	-0,0100%	-0,0417%	-0,1384%	-0,3414%	-0,8362 %
$\sigma=30\%$	-0,0024%	-0,0120%	-0,0511%	-0,1662%	-0,4059%	-0,9939 %
$\sigma=40\%$	-0,0032%	-0,0159%	-0,0675%	-0,2304%	-0,5940%	-1,4318 %
$\mu=15\%$						
$\sigma=20\%$	-0,0016%	-0,0082%	-0,0369%	-0,1394%	-0,4001%	-1,2030%
$\sigma=25\%$	-0,0020%	-0,0102%	-0,0458%	-0,1707%	-0,4818%	-1,5083%
$\sigma=30\%$	-0,0024%	-0,0124%	-0,0542%	-0,2065%	-0,5887%	-1,7691%
$\sigma=40\%$	-0,0032%	-0,0163%	-0,0724%	-0,2915%	-0,7707%	-2,4531%
$\mu=20\%$						
$\sigma=20\%$	-0,0016%	-0,0085%	-0,0411%	-0,1842%	-0,5808%	-2,0035%
$\sigma=25\%$	-0,0020%	-0,0106%	-0,0509%	-0,2185%	-0,7138%	-2,3628%
$\sigma=30\%$	-0,0024%	-0,0128%	-0,0599%	-0,2588%	-0,8281%	-2,7345%
$\sigma=40\%$	-0,0032%	-0,0170%	-0,0808%	-0,3576%	-1,0598%	-3,6156%
$\mu=30\%$						
$\sigma=20\%$	-0,0016%	-0,0091%	-0,0543%	-0,3110%	-1,1808%	-4,3914%
$\sigma=25\%$	-0,0020%	-0,0115%	-0,0656%	-0,3850%	-1,3231%	-4,8761%
$\sigma=30\%$	-0,0024%	-0,0135%	-0,0758%	-0,4115%	-1,4971%	-5,5945%
$\sigma=40\%$	-0,0032%	-0,0179%	-0,0996%	-0,5295%	-1,9042%	-6,9516%

$r=0.05$, $f = 0$, spread=0.00316

Table 7.3-Comparison of M-squared measures: M2U-M2V=M2D

M2D	Daily	Weekly	Monthly	Quarterly	Semi-Annually	Annually
T	0,004	0,020	0,080	0,240	0,500	1,000
$\mu=10\%$						
$\sigma=20\%$	0,0016%	0,0079%	0,0318%	0,0951%	0,1968%	0,3887%
$\sigma=25\%$	0,0020%	0,0099%	0,0397%	0,1184%	0,2440%	0,4785%
$\sigma=30\%$	0,0024%	0,0118%	0,0476%	0,1415%	0,2900%	0,5592%
$\sigma=40\%$	0,0032%	0,0158%	0,0632%	0,1865%	0,3768%	0,7048%
$\mu=15\%$						
$\sigma=20\%$	0,0016%	0,0080%	0,0317%	0,0940%	0,1915%	0,3697%
$\sigma=25\%$	0,0020%	0,0099%	0,0396%	0,1169%	0,2381%	0,4517%
$\sigma=30\%$	0,0024%	0,0119%	0,0474%	0,1399%	0,2823%	0,5355%
$\sigma=40\%$	0,0032%	0,0159%	0,0630%	0,1840%	0,3672%	0,6762%
$\mu=20\%$						
$\sigma=20\%$	0,0016%	0,0080%	0,0315%	0,0927%	0,1874%	0,3510%
$\sigma=25\%$	0,0020%	0,0099%	0,0394%	0,1156%	0,2315%	0,4322%
$\sigma=30\%$	0,0024%	0,0119%	0,0473%	0,1381%	0,2759%	0,5072%
$\sigma=40\%$	0,0032%	0,0158%	0,0628%	0,1820%	0,3588%	0,6428%
$\mu=30\%$						
$\sigma=20\%$	0,0016%	0,0079%	0,0313%	0,0903%	0,1771%	0,3162%
$\sigma=25\%$	0,0020%	0,0100%	0,0391%	0,1397%	0,2202%	0,3901%
$\sigma=30\%$	0,0024%	0,0119%	0,0469%	0,1346%	0,2615%	0,4576%
$\sigma=40\%$	0,0032%	0,0158%	0,0622%	0,1775%	0,3407%	0,5808%

$r=0.05$, $f = 0$, spread=0.00316

Table 7.4-Estimated daily expected returns of U and V.

U	Low	Estimate	High	V	Low	Estimate	High
Daily T=0.004	U			Daily T=0.004	V		
$\mu=10\%$				$\mu=10\%$			
$\sigma=20\%$	0,0585%	0,0604%	0,0622%	$\sigma=20\%$	0,0553%	0,0572%	0,0591%
$\sigma=25\%$	0,0569%	0,0592%	0,0616%	$\sigma=25\%$	0,0529%	0,0552%	0,0576%
$\sigma=30\%$	0,0560%	0,0588%	0,0617%	$\sigma=30\%$	0,0512%	0,0541%	0,0569%
$\sigma=40\%$	0,0554%	0,0591%	0,0629%	$\sigma=40\%$	0,0490%	0,0528%	0,0565%
$\mu=15\%$				$\mu=15\%$			
$\sigma=20\%$	0,0969%	0,0987%	0,1006 %	$\sigma=20\%$	0,0937 %	0,0955%	0,0974%
$\sigma=25\%$	0,0993%	0,1016%	0,1039 %	$\sigma=25\%$	0,0953 %	0,0976%	0,1000%
$\sigma=30\%$	0,0992%	0,1020%	0,1049 %	$\sigma=30\%$	0,0944 %	0,0973%	0,1001%
$\sigma=40\%$	0,0966%	0,1004%	0,1041 %	$\sigma=40\%$	0,0903 %	0,0940%	0,0978%
$\mu=20\%$				$\mu=20\%$			
$\sigma=20\%$	0,1367%	0,1411%	0,1429%	$\sigma=20\%$	0,1360%	0,1379%	0,1392%
$\sigma=25\%$	0,1379%	0,1402%	0,1426%	$\sigma=25\%$	0,1339%	0,1362%	0,1386%
$\sigma=30\%$	0,1360%	0,1388%	0,1417%	$\sigma=30\%$	0,1312%	0,1341%	0,1369%
$\sigma=40\%$	0,1374%	0,1411%	0,1449%	$\sigma=40\%$	0,1310%	0,1348%	0,1385%
$\mu=30\%$				$\mu=30\%$			
$\sigma=20\%$	0,2174%	0,2193%	0,2211%	$\sigma=20\%$	0,2142%	0,2161%	0,2179%
$\sigma=25\%$	0,2197%	0,2221%	0,2244%	$\sigma=25\%$	0,2157%	0,2181%	0,2204%
$\sigma=30\%$	0,2191%	0,2220%	0,2248%	$\sigma=30\%$	0,2144%	0,2172%	0,2200%
$\sigma=40\%$	0,2161%	0,2198%	0,2236%	$\sigma=40\%$	0,2097%	0,2134%	0,2172%

$r=0.05$, $f = 0$, spread=0.00316

Table 7.5-Estimated weekly expected returns of U and V.

U	Low	Estimate	High	V	Low	Estimate	High
Weekly T=0.02	U			Weekly T=0.02	V		
$\mu=10\%$				$\mu=10\%$			
$\sigma=20\%$	0,2885%	0,2950%	0,3014%	$\sigma=20\%$	0,2726%	0,2790%	0,2854%
$\sigma=25\%$	0,2936%	0,3016%	0,3097%	$\sigma=25\%$	0,2736%	0,2816%	0,2897%
$\sigma=30\%$	0,2907%	0,3004%	0,3100%	$\sigma=30\%$	0,2667%	0,2764%	0,2860%
$\sigma=40\%$	0,2807%	0,2936%	0,3064%	$\sigma=40\%$	0,2487%	0,2616%	0,2744%
$\mu=15\%$				$\mu=15\%$			
$\sigma=20\%$	0,4923%	0,4987%	0,5052%	$\sigma=20\%$	0,4763%	0,4827%	0,4891%
$\sigma=25\%$	0,4929%	0,5009%	0,5089%	$\sigma=25\%$	0,4728%	0,4809%	0,4889%
$\sigma=30\%$	0,4947%	0,5043%	0,5140%	$\sigma=30\%$	0,4707%	0,4803%	0,4900%
$\sigma=40\%$	0,4741%	0,4870%	0,4998%	$\sigma=40\%$	0,4421%	0,4549%	0,4678%
$\mu=20\%$				$\mu=20\%$			
$\sigma=20\%$	0,6959%	0,7023%	0,7088%	$\sigma=20\%$	0,6798%	0,6863%	0,6927%
$\sigma=25\%$	0,6926%	0,7006%	0,7087%	$\sigma=25\%$	0,6725%	0,6806%	0,6886%
$\sigma=30\%$	0,6863%	0,6959%	0,7056%	$\sigma=30\%$	0,6622%	0,6719%	0,6815%
$\sigma=40\%$	0,6917%	0,7046%	0,7176%	$\sigma=40\%$	0,6596%	0,6726%	0,6855%
$\mu=30\%$				$\mu=30\%$			
$\sigma=20\%$	1,0958%	1,1023%	1,1088%	$\sigma=20\%$	1,0797%	1,0862%	1,0926%
$\sigma=25\%$	1,0927%	1,1008%	1,1088%	$\sigma=25\%$	1,0725%	1,0806%	1,0887%
$\sigma=30\%$	1,0938%	1,1035%	1,1132%	$\sigma=30\%$	1,0696%	1,0793%	1,0890%
$\sigma=40\%$	1,0924%	1,1054%	1,1183%	$\sigma=40\%$	1,0602%	1,0732%	1,0861%

$r=0.05$, $f = 0$, spread=0.00316

Table 7.6-Estimated monthly expected returns of U and V.

U	Low	Estimate	High	V	Low	Estimate	High
Monthly T=0.08	U			Monthly T=0.08	V		
$\mu=10\%$				$\mu=10\%$			
$\sigma=20\%$	1,1663%	1,1901%	1,2138%	$\sigma=20\%$	1,1018%	1,1255%	1,1492%
$\sigma=25\%$	1,1736%	1,2033%	1,2331%	$\sigma=25\%$	1,1164%	1,1460%	1,1557%
$\sigma=30\%$	1,1545%	1,1902%	1,2259%	$\sigma=30\%$	1,0930%	1,1227%	1,1524%
$\sigma=40\%$	1,1310%	1,1788%	1,2266%	$\sigma=40\%$	1,0021%	1,0499%	1,0976%
$\mu=15\%$				$\mu=15\%$			
$\sigma=20\%$	2,0090%	2,0248%	2,0487%	$\sigma=20\%$	1,9358%	1,9597%	1,9836%
$\sigma=25\%$	1,9926%	2,0225%	2,0525%	$\sigma=25\%$	1,9113%	1,9412%	1,9712%
$\sigma=30\%$	2,0051%	2,0411%	2,0771%	$\sigma=30\%$	1,9076%	1,9436%	1,9745%
$\sigma=40\%$	2,0038%	2,0520%	2,1002%	$\sigma=40\%$	1,8738%	1,9220%	1,9701%
$\mu=20\%$				$\mu=20\%$			
$\sigma=20\%$	2,8351%	2,8592%	2,8833%	$\sigma=20\%$	2,7695%	2,7936%	2,8177%
$\sigma=25\%$	2,8209%	2,8510%	2,8812%	$\sigma=25\%$	2,7390%	2,7691%	2,7992%
$\sigma=30\%$	2,8051%	2,8414%	2,8776%	$\sigma=30\%$	2,7068%	2,7430%	2,7793%
$\sigma=40\%$	2,8066%	2,8552%	2,9038%	$\sigma=40\%$	2,6756%	2,7242%	2,7727%
$\mu=30\%$				$\mu=30\%$			
$\sigma=20\%$	4,4710%	4,4955%	4,5200%	$\sigma=20\%$	4,4042%	4,4287%	4,4532%
$\sigma=25\%$	4,4606%	4,4913%	4,5219%	$\sigma=25\%$	4,3773%	4,4079%	4,4386%
$\sigma=30\%$	4,4567%	4,4935%	4,5304%	$\sigma=30\%$	4,3568%	4,3936%	4,4304%
$\sigma=40\%$	4,4136%	4,4630%	4,5123%	$\sigma=40\%$	4,2806%	4,3299%	4,3792%

$r=0.05$, $f = 0$, spread=0.00316

Table 7.7-Estimated daily standard deviations of U and V.

stdU	Low	Estimate	High	stdV	Low	Estimate	High
Daily T=0.004	U			Daily T=0.004	V		
$\mu=10\%$				$\mu=10\%$			
$\sigma=20\%$	2,5287%	2,5300%	2,5313%	$\sigma=20\%$	2,5285%	2,5298%	2,5312%
$\sigma=25\%$	3,1623%	3,1640%	3,1656%	$\sigma=25\%$	3,1622%	3,1638%	3,1655%
$\sigma=30\%$	3,7943%	3,7963%	3,7983%	$\sigma=30\%$	3,7941%	3,7961%	3,7981%
$\sigma=40\%$	5,0582%	5,0608%	5,0635%	$\sigma=40\%$	5,0579%	5,0605%	5,0632%
$\mu=15\%$				$\mu=15\%$			
$\sigma=20\%$	2,5300%	2,5313%	2,5327%	$\sigma=20\%$	2,5298%	2,5312%	2,5325%
$\sigma=25\%$	3,1629%	3,1645%	3,1662%	$\sigma=25\%$	3,1627%	3,1643%	3,1660%
$\sigma=30\%$	3,7953%	3,7973%	3,7992%	$\sigma=30\%$	3,7950%	3,7970%	3,7990%
$\sigma=40\%$	5,0603%	5,0629%	5,0656%	$\sigma=40\%$	5,0600%	5,0626%	5,0653%
$\mu=20\%$				$\mu=20\%$			
$\sigma=20\%$	2,5299%	2,5312%	2,5326%	$\sigma=20\%$	2,5297%	2,5310%	2,5323%
$\sigma=25\%$	3,1619%	3,1635%	3,1652%	$\sigma=25\%$	3,1616%	3,1633%	3,1649%
$\sigma=30\%$	3,7943%	3,7963%	3,7983%	$\sigma=30\%$	3,7940%	3,7960%	3,7980%
$\sigma=40\%$	5,0629%	5,0655%	5,0682%	$\sigma=40\%$	5,0625%	5,0652%	5,0678%
$\mu=30\%$				$\mu=30\%$			
$\sigma=20\%$	2,5301%	2,5314%	2,5328%	$\sigma=20\%$	2,5298%	2,5311%	2,5324%
$\sigma=25\%$	3,1641%	3,1657%	3,1674%	$\sigma=25\%$	3,1637%	3,1654%	3,1670%
$\sigma=30\%$	3,7973%	3,7993%	3,8013%	$\sigma=30\%$	3,7969%	3,7989%	3,8009%
$\sigma=40\%$	5,0638%	5,0665%	5,0691%	$\sigma=40\%$	5,0634%	5,0660%	5,0687%

$r=0.05$, $f = 0$, spread=0.00316

Table 7.8-Estimated weekly standard deviations of U and V.

stdU	Low	Estimate	High	stdV	Low	Estimate	High
Weekly T=0.02	U			Weekly T=0.02	V		
$\mu=10\%$				$\mu=10\%$			
$\sigma=20\%$	5,6702%	5,6747%	5,6793%	$\sigma=20\%$	5,6691%	5,6737%	5,6782%
$\sigma=25\%$	7,0915%	7,0972%	7,1028%	$\sigma=25\%$	7,0900%	7,0957%	7,1014%
$\sigma=30\%$	8,5085%	8,5153%	8,5221%	$\sigma=30\%$	8,5065%	8,5133%	8,5201%
$\sigma=40\%$	11,3705%	11,3796%	11,3887%	$\sigma=40\%$	11,3671%	11,3762%	11,3853%
$\mu=15\%$				$\mu=15\%$			
$\sigma=20\%$	5,6806%	5,6852%	5,6897%	$\sigma=20\%$	5,7695%	5,6840%	5,6886%
$\sigma=25\%$	7,1056%	7,1113%	7,1170%	$\sigma=25\%$	7,1040%	7,1097%	7,1154%
$\sigma=30\%$	8,5342%	8,5411%	8,5479%	$\sigma=30\%$	8,5321%	8,5389%	8,5457%
$\sigma=40\%$	11,3805%	11,3896%	11,3987%	$\sigma=40\%$	11,3769%	11,3860%	11,3987%
$\mu=20\%$				$\mu=20\%$			
$\sigma=20\%$	5,6907%	5,6952%	5,6998%	$\sigma=20\%$	5,6894%	5,6940%	5,6985%
$\sigma=25\%$	7,1183%	7,1240%	7,1297%	$\sigma=25\%$	7,1166%	7,1223%	7,1280%
$\sigma=30\%$	8,5411%	8,5480%	8,5548%	$\sigma=30\%$	8,5389%	8,5457%	8,5525%
$\sigma=40\%$	11,3975%	11,4066%	11,4158%	$\sigma=40\%$	11,3938%	11,4030%	11,4121%
$\mu=30\%$				$\mu=30\%$			
$\sigma=20\%$	5,7097%	5,7143%	5,7188%	$\sigma=20\%$	5,7082%	5,7128%	5,7173%
$\sigma=25\%$	7,1398%	7,1455%	7,1513%	$\sigma=25\%$	7,1379%	7,1436%	7,1493%
$\sigma=30\%$	8,5776%	8,5845%	8,5913%	$\sigma=30\%$	8,5751%	8,5819%	8,5888%
$\sigma=40\%$	11,4504%	11,4596%	11,4688%	$\sigma=40\%$	11,4465%	11,4556%	11,4648%

$r=0.05$, $f = 0$, spread=0.00316

Table 7.9-Estimated monthly standard deviations of U and V.

stdU	Estimate			stdV	Estimate		
Monthly T=0.08	Low	High		Monthly T=0.08	Low	High	
U				V			
$\mu=10\%$				$\mu=10\%$			
$\sigma=20\%$	11,4775%	11,4943%	11,5111%	$\sigma=20\%$	11,4700%	11,4867%	11,5035%
$\sigma=25\%$	14,3586%	14,3796%	14,4007%	$\sigma=25\%$	14,3471%	14,3680%	14,3891%
$\sigma=30\%$	17,2877%	17,3129%	17,3383%	$\sigma=30\%$	17,2712%	17,2964%	17,3217%
$\sigma=40\%$	23,1289%	23,1627%	23,1965%	$\sigma=40\%$	23,0998%	23,1336%	23,1674%
$\mu=15\%$				$\mu=15\%$			
$\sigma=20\%$	11,5646%	11,5815%	11,5985%	$\sigma=20\%$	11,5568%	11,5736%	11,5906%
$\sigma=25\%$	14,4771%	14,4982%	14,5194%	$\sigma=25\%$	14,4651%	14,4863%	14,5075%
$\sigma=30\%$	17,3884%	17,4138%	17,4393%	$\sigma=30\%$	17,3716%	17,3969%	17,4224%
$\sigma=40\%$	23,3011%	23,3352%	23,3693%	$\sigma=40\%$	23,2716%	23,3056%	23,3396%
$\mu=20\%$				$\mu=20\%$			
$\sigma=20\%$	11,6407%	11,6577%	11,6747%	$\sigma=20\%$	11,6326%	11,6496%	11,6666%
$\sigma=25\%$	14,5911%	14,6124%	14,6338%	$\sigma=25\%$	14,5789%	14,6002%	14,6215%
$\sigma=30\%$	17,5066%	17,5322%	17,5578%	$\sigma=30\%$	17,4894%	17,5150%	17,5406%
$\sigma=40\%$	23,5341%	23,5685%	23,6030%	$\sigma=40\%$	23,5040%	23,5383%	23,5728%
$\mu=30\%$				$\mu=30\%$			
$\sigma=20\%$	11,8382%	11,8555%	11,8728%	$\sigma=20\%$	11,8294%	11,8467%	11,8640%
$\sigma=25\%$	14,8124%	14,8341%	14,8558%	$\sigma=25\%$	14,7995%	14,8211%	14,8428%
$\sigma=30\%$	17,7986%	17,8246%	17,8506%	$\sigma=30\%$	17,7806%	17,8066%	17,8326%
$\sigma=40\%$	23,8490%	23,8839%	23,9188%	$\sigma=40\%$	23,8180%	23,8528%	23,8877%

$r=0.05$, $f = 0$, spread=0.00316

Table 7.10-Estimated daily, weekly and monthly double standard deviation of S.

SsdX2	Low	Estimate	High	SsdX2	Low	Estimate	High	SsdX2	Low	Estimate	High
Daily T=0.004				Weekly T=0.02				Monthly T=0.08			
$\mu=10\%$				$\mu=10\%$				$\mu=10\%$			
$\sigma=20\%$	2,5285%	2,5300%	2,5313%	$\sigma=20\%$	5,6660%	5,6706%	5,6751%	$\sigma=20\%$	11,3956%	11,4122%	11,4289%
$\sigma=25\%$	3,1613%	3,1630%	3,1646%	$\sigma=25\%$	7,0853%	7,0910%	7,0967%	$\sigma=25\%$	14,2599%	14,2808%	14,3017%
$\sigma=30\%$	3,7940%	3,7960%	3,7980%	$\sigma=30\%$	8,4953%	8,5021%	8,5090%	$\sigma=30\%$	17,1055%	17,1305%	17,1555%
$\sigma=40\%$	5,0588%	5,0615%	5,0641%	$\sigma=40\%$	11,3358%	11,3449%	11,3540%	$\sigma=40\%$	22,8291%	22,8624%	22,8959%
$\mu=15\%$				$\mu=15\%$				$\mu=15\%$			
$\sigma=20\%$	2,5299%	2,5313%	2,5326%	$\sigma=20\%$	5,6719%	5,6764%	5,6809%	$\sigma=20\%$	11,4536%	11,4703%	11,4871%
$\sigma=25\%$	3,1633%	3,1650%	3,1667%	$\sigma=25\%$	7,0867%	7,0924%	7,0980%	$\sigma=25\%$	14,3244%	14,3454%	14,3663%
$\sigma=30\%$	3,7953%	3,7973%	3,7993%	$\sigma=30\%$	8,5122%	8,5190%	8,5258%	$\sigma=30\%$	17,1623%	17,1875%	17,2125%
$\sigma=40\%$	5,0623%	5,0650%	5,0676%	$\sigma=40\%$	11,3510%	11,3601%	11,3692%	$\sigma=40\%$	22,9377%	22,9713%	23,0049%
$\mu=20\%$				$\mu=20\%$				$\mu=20\%$			
$\sigma=20\%$	2,5305%	2,5318%	2,5332%	$\sigma=20\%$	5,6793%	5,6838%	5,6884%	$\sigma=20\%$	11,4887%	11,5055%	11,5223%
$\sigma=25\%$	3,1629%	3,1645%	3,1666%	$\sigma=25\%$	7,0921%	7,0977%	7,1034%	$\sigma=25\%$	14,3623%	14,3833%	14,4044%
$\sigma=30\%$	3,7956%	3,7976%	3,7996%	$\sigma=30\%$	8,5112%	8,5180%	8,5248%	$\sigma=30\%$	17,2464%	17,2716%	17,2968%
$\sigma=40\%$	5,0639%	5,0666%	5,0693%	$\sigma=40\%$	11,3634%	11,3725%	11,3816%	$\sigma=40\%$	23,0177%	23,0513%	23,0850%
$\mu=30\%$				$\mu=30\%$				$\mu=30\%$			
$\sigma=20\%$	2,5310%	2,5323%	2,5336%	$\sigma=20\%$	5,6883%	5,6928%	5,6974%	$\sigma=20\%$	11,5768%	11,5937%	11,6106%
$\sigma=25\%$	3,1635%	3,1652%	3,1668%	$\sigma=25\%$	7,1096%	7,1147%	7,1204%	$\sigma=25\%$	14,4959%	14,5170%	14,5383%
$\sigma=30\%$	3,7974%	3,7994%	3,8014%	$\sigma=30\%$	8,5302%	8,5370%	8,5438%	$\sigma=30\%$	17,4053%	17,4307%	17,4562%
$\sigma=40\%$	5,0696%	5,0699%	5,0643%	$\sigma=40\%$	11,3836%	11,3927%	11,4018%	$\sigma=40\%$	23,2442%	23,2781%	23,3122%

$r=0.05, f = 0, \text{spread}=0.00316$

Table 7.11-Estimated daily expected returns with management fee and bid-ask spread.

V^*	Estimate		
Daily $T=0.004$	Low	Estimate	High
$\mu=10\%$			
$\sigma=20\%$	0,0508%	0,0526%	0,0545%
$\sigma=25\%$	0,0518%	0,0541%	0,0565%
$\sigma=30\%$	0,0482%	0,0510%	0,0538%
$\sigma=40\%$	0,0474%	0,0511%	0,0549%
$\mu=15\%$			
$\sigma=20\%$	0,0926%	0,0945%	0,0963%
$\sigma=25\%$	0,0901%	0,0925%	0,0948%
$\sigma=30\%$	0,0869%	0,0897%	0,0925%
$\sigma=40\%$	0,0840%	0,0877%	0,0915%
$\mu=20\%$			
$\sigma=20\%$	0,1309%	0,1328%	0,1346%
$\sigma=25\%$	0,1308%	0,1332%	0,1355%
$\sigma=30\%$	0,1299%	0,1327%	0,1356%
$\sigma=40\%$	0,1262%	0,1300%	0,1337%
$\mu=30\%$			
$\sigma=20\%$	0,2116%	0,2135%	0,2153%
$\sigma=25\%$	0,2102%	0,2125%	0,2149%
$\sigma=30\%$	0,2069%	0,2097%	0,2125%
$\sigma=40\%$	0,2051%	0,2088%	0,2126%

$r=0.05$, $f = 0.008$, spread=0.00316

Table 7.12-Estimated daily expected return of S.

S , $r=5\%$	Low	Estimate	High
Weekly $T=0.02$			
$\mu=10\%$			
$\sigma=20\%$	0,3914%	0,3978%	0,4042%
$\sigma=25\%$	0,3925%	0,4005%	0,4085%
$\sigma=30\%$	0,3901%	0,3998%	0,4094%
$\sigma=40\%$	0,3958%	0,4087%	0,4215%

$r=0.05$, $f = 0$, spread=0.00316

Table 7.13-Estimated daily expected return of U, V and estimated daily double expected return of S.

r=0%	Low	Estimate	High
Weekly T=0.02			
$\mu=10\%$	V		
$\sigma=20\%$	0,3796%	0,3860%	0,3924%
$\sigma=25\%$	0,3777%	0,3857%	0,3938%
$\sigma=30\%$	0,3640%	0,3737%	0,3833%
$\sigma=40\%$	0,3524%	0,3653%	0,3782%
$\mu=10\%$		U	
$\sigma=20\%$	0,3956%	0,4020%	0,4084%
$\sigma=25\%$	0,3977%	0,4057%	0,4138%
$\sigma=30\%$	0,3880%	0,3977%	0,4073%
$\sigma=40\%$	0,3844%	0,3973%	0,4102%
$\mu=10\%$		S	
$\sigma=20\%$	0,3953%	0,4018%	0,4082%
$\sigma=25\%$	0,3973%	0,4053%	0,4134%
$\sigma=30\%$	0,3878%	0,3974%	0,4070%
$\sigma=40\%$	0,3840%	0,3969%	0,4097%

$r=0.05, f = 0, \text{spread}=0.00316$